APPLIED MECHANICS Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

REVS. 4042-4356 VOL. 4, NO. 11 NOVEMBER 1951

Published Monthly by THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS at Easton, Pa., and edited by Midwest Research Institute with the co-operation of Linda Hall Library

APPLIED MECHANICS *Reviews*

Editorial Office: APPLIED MECHANICS REVIEWS, Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Mo., U.S.A.

Subscription and Production Office: The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., U.S.A.

HOW TO OBTAIN COPIES OF ARTICLES INDEXED: Photocopy or microfilm copies of all articles reviewed in this issue will be provided on request. Orders should specify the number and issue of APPLIED MECHANICS REVIEWS; should be addressed to APPLIED MECHANICS REVIEWS, Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Mo., and be accompanied by a remittance to cover costs. Photocopy costs are 25¢ for each page of the article photocopied; minimum charge $1; Microfilm costs include service charge of 50¢ per article, plus 3¢ per double page; minimum order, 75¢. *(Applicant assumes responsibility for questions of copyright arising from this copying and the use made of copies. Copyright material will not be reproduced beyond recognized fair use without consent of copyright owner.)*

APPLIED MECHANICS REVIEWS, November 1951, Vol. 4, No. 11. Published monthly by The American Society of Mechanical Engineers at 20th and Northampton St., Easton, Pa., U. S. A. The editorial office is located at the Midwest Research Institute, Kansas City 2, Mo., U. S. A. Headquarters of ASME, 29 West 39th St., New York 18, N. Y. Cable address: "Dynamic," New York. Price $1.50 per copy, $12.50 a year, to members of ASME and co-operating societies $0.75 per copy, $9 a year. Extra postage to countries not in the Pan-American Union, $1.50; to Canada, $0.75. Changes of address must be received at Society headquarters four weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publication (B13, Par. 4). Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. Copyrighted, 1951, by the American Society of Mechanical Engineers.

APPLIED MECHANICS REVIEWS

VOL. 4, NO. 11 — MARTIN GOLAND *Editor* — NOVEMBER 1951

RECENT CONTRIBUTIONS TO THE THEORY OF PLASTICITY

WILLIAM PRAGER

PROFESSOR OF APPLIED MECHANICS, BROWN UNIVERSITY, PROVIDENCE, R. I.

IN THIS survey, the term "theory of plasticity" is used in the sense commonly accepted in engineering. This theory is concerned with the analysis of stresses and strains in plastic solids; it uses the general methods of mechanics of continua in conjunction with special stress-strain laws purporting to give a phenomenological description of the plastic behavior of the solids under consideration. The physicist may well express the opinion that an investigation of this kind has the nature of a mathematical exercise which sheds no light on the basic mechanism of plastic deformation and should, therefore, not be dignified by the term "theory." The engineer, on the other hand, can point to the analogy with the well-established use of the term "theory of elasticity."

The theory of plasticity was founded by B. de Saint-Venant in 1870 (*C. R. Acad. Sci. Paris* **70**, 473–480, 1870). In its history, periods of rapid progress alternate with periods of comparative stagnation. The progress made in the past five years has been considerable and seems worth reviewing at this time.

The state of the field at the beginning of the period under consideration is admirably presented in V. V. Sokolovsky's book "Theory of Plasticity," in Russian with English chapter summaries, Moscow, 1946). In the following brief review of recent progress in various branches of the mathematical theory of plasticity, the state at the beginning of the review period will therefore occasionally be indicated by reference to Sokolovsky's book. At the time of its publication, this work represented the most nearly complete exposition of the theory available in any language. The progress made since then is most strikingly illustrated by the fact that, after only a few years, this work is no longer an adequate introduction to the current literature.*

1 TEXTBOOKS

Recent textbooks on the mathematical theory of plasticity or closely related fields are: van Iterson (1947, AMR **2**, Rev. 47), Colonnetti (1948, AMR **3**, Rev. 875), Ilyushin (1948, AMR **4**, Rev. 2913), Van den Broek (1948, AMR **2**, Rev. 1131), Reiner (1949, AMR **4**, Revs. 1158, 1159), Freudenthal (1950, AMR **3**, Rev. 1922), Hill (1950, AMR **4**, Rev. 2471), Nadai (1950, AMR **4**, Rev. 2470), Saliger (1950, AMR **4**, Rev. 663), and Prager and Hodge (Wiley, 1951). In the following, reference to books from this list will be made by giving the author's name and the page number.

2 STRESS-STRAIN LAWS

In Sokolovsky's book the "total" or "finite strain theory" (or "deformation theory") of Hencky (*Z. angew. Math. Mech.* **4**, 323–334, 1924) and the "incremental strain theory" (or "flow theory") of von Mises (*Goettinger Nachrichten, math.-phys. Kl.* **1913**, 582–592, 1913) are discussed side by side, the choice of one or the other being treated as a question of mathematical expediency. Following Ilyushin (*Prikl. Mat. Mekh.* **9**, 207–218, 1945; **10**, 347–356, 1946) these two types of stress-strain law have been widely discussed for perfectly plastic as well as work-hardening materials. Handelman, Lin, and Prager (AMR **1**, Revs. 99, 274, 1122) have shown that even the most general total strain theory fails to satisfy certain continuity conditions at the boundary between the elastic and plastic ranges. The argument of these authors has been given a more incisive form by Drucker (AMR **3**, Revs. 1087, 1938, 2300) and Hill (p. 47).

While a stress-strain law of the deformation type cannot possibly describe the complete plastic behavior of a given material, it may furnish an adequate description for special programs of loading. For instance, when a specimen is strained in such a manner that all strain components increase in strict proportion, each total strain theory can be matched with an incremental strain theory which predicts the same stresses throughout this process of "simple loading" (Ilyushin, AMR **2**, Rev. 733). Unfortunately, most experiments in the plastic range have been conducted under conditions of simple loading and hence do not provide an empirical refutation of one or the other type of theory. Experiments with more general loading programs have been reported by Dorn and Thomsen (AMR **1**, Rev 833), Fraenkel (AMR **1**, Rev. 1467), Peters, Dow, and Batdorf (AMR **3**, Rev. 2297), Stüssi (AMR **4**, Rev. 1147), Morrison and Shepherd (AMR **4**, Rev. 2488), and Marin, Ulrich, and Hughes (*Nat. adv. Comm. Aero. tech. note* 2425). While the results of such experiments are not in full agreement with the simplest incremental strain theory formulated by Prandtl (Proc. First Internat. Congr. Appl. Mech., Delft 1924, pp. 42–54) and Reuss (*Z. angew. Math.*

* The recently published second edition (Moscow, 1950) contains only minor changes aside from some rearrangement of the material.

EDITOR'S NOTE: The eleventh in a series of articles, contributed to APPLIED MECHANICS REVIEWS by international authorities, surveying important topics in applied mechanics.

Mech. **10**, 266–274, 1930), they are, on the whole, in better agreement with it than with the corresponding total strain theory. Ways of fitting more complex incremental strain theories to experimental results have been discussed by Prager (AMR **1**, Rev. 455) and Drucker, Stockton, and White (AMR **3**, Revs. 2301, 2302; **4**, Rev. 2473).

In the older literature, the yield condition and the relation between stresses and increments of plastic strain are treated as independent ingredients of a stress-strain law. An interdependence of these ingredients has been stipulated by von Mises (*Z. angew. Math. Mech.* **8**, 161–185, 1928), Taylor (AMR **1**, Rev. 277), and Hill, Lee, and Tupper (AMR **1**, Rev. 282). This theory of the plastic potential can now be considered as firmly established for perfectly plastic as well as work-hardening materials by the work of Melan (*Ing.-Arch.* **9**, 116–126, 1938), Prager and Hodge (AMR **2**, Revs. 1003, 1387), Drucker (AMR **3**, Revs. 1691, 1920), and Edelman and Drucker (AMR **4**, Rev. 3865). The essential feature of this theory can be stated in geometric terms as follows (see Prager, AMR **2**, Rev. 1003). If only isothermal deformations are considered, the state of a plastic material is defined by the strain history. A given state is called elastic if any infinitesimal change of stress leaves the permanent strain unchanged; if this condition is not fulfilled, the considered state is called plastic. The elastic range associated with a given state is defined as the totality of all elastic states which can be reached from the given state without passing through plastic states. The yield limit of an elastic range is the totality of the stress tensors of the plastic states which bound this range. It is often convenient to represent the yield limit by a surface ("yield surface") in a suitably defined stress space. A generic point of this surface represents a state of stress at the yield limit. Under this state of stress a change of permanent strain is possible which will be infinitesimal in the case of a work-hardening material, but may be finite for a perfectly plastic material. Let this change of permanent strain be represented by a vector such that the mechanical energy dissipated during this change of strain is given by the scalar product of this vector and the radius vector of the considered point on the yield surface. If then this surface has continuously turning tangent plane, the vector representing the change of permanent strain has the direction of the exterior normal to the yield surface at the point under consideration. If the yield surface has edges or corners, this rule must be suitably supplemented for states of stress represented by points on edges or by corners.

An incremental strain theory of a novel type has been proposed by Batdorf and Budiansky (AMR **2**, Rev. 1264). The equations of the "slip theory" are based on a physical model. In view of the many idealizations and arbitrary assumptions incorporated in this model, the success of this theory is not likely to be due to this "physical basis." In the geometric terms discussed above, the really important feature of the slip theory seems to be the following: During work-hardening the yield surface changes its shape in such a manner that the point representing the instantaneous state of stress always forms a corner of this surface (Budiansky, Ph.D. thesis, Brown University, 1950). The possibility of a yield surface of this singular type had been overlooked in earlier discussions. Other implications of the slip theory have been discussed by Cicala (AMR **4**, Revs. 2033, 2482).

Because important structural elements such as cold-rolled sheets and extruded sections lack isotropy, stress-strain laws for anisotropic plastic materials have been receiving an increasing amount of attention. Such laws have been discussed by Hill (AMR **1**, Rev. 1121; **3**, Rev. 2650), Dorn (AMR **2**, Rev. 866), Fisher (AMR **3**, Rev. 1264), and Hazlett, Robinson, and Dorn (AMR **3**, Rev. 2296).

A comprehensive survey entitled "Stress-strain relations in the plastic range, a survey of theory and experiment," by D. C. Drucker has been published by Brown University as Report A11-S1 to ONR, Dec. 1950 (AMR **4**, Rev. 3553). A brief survey of stress-strain laws has been given by Prager (AMR **1**, Rev. 1466).

3 EXTREMUM PRINCIPLES

In the theory of elasticity the principles of minimum potential energy and minimum complementary energy are widely used to obtain approximate solutions. Since it seems likely that extremum principles will prove equally useful in the theory of plasticity, it is not surprising that these principles have been studied intensively in recent years. An older principle of Colonnetti (see, for instance, *J. Math. pures appl.* (9) **17**, 223–255, 1938, and AMR **3**, Rev. 1860) has been applied by Pizetti (AMR **3**, Rev. 61). New extremum principles have been established by Markov (AMR **1**, Rev. 97), Hill (AMR **1**, Rev. 1354), and Greenberg (AMR **2**, Rev. 1265) for perfectly plastic materials, and by Hodge and Prager (AMR **2**, Rev. 1387), and Hill (pp. 63–64) for work-hardening materials. Phillips (AMR **2**, Rev. 475; **3**, Rev. 262) has discussed the variational principles for certain total strain theories.

A comprehensive survey of the variational principles of plasticity has been given by H. J. Greenberg (AMR **3**, Rev. 466). A brief comparative study of these principles has been given by Hill (AMR **4**, Rev. 198).

4 LIMIT ANALYSIS

It has long been known that stress concentrations predicted by the theory of elasticity are greatly alleviated by ductility, and that elastic stress concentration factors do not provide a sound basis for judging the load-carrying capacity of structural elements made of ductile materials. Elastic design of an indeterminate structure yields also too low a value of the load-carrying capacity. The terms "limit design" or "limit analysis" are used to indicate the application of the theory of plasticity to a more realistic determination of the load-carrying capacity. Work along these lines was started independently by Kazinczy in Hungary (1914) and Kist in Holland (1917); the first applications were to continuous beams. Van den Broek's book (AMR **2**, Rev. 1131) contains many illuminating examples of this type. The last few years brought a rapid expansion of the literature in this field. Beam problems have been discussed by Hrennikoff (AMR **1**, Rev. 55), Panlilio (AMR **1**, Rev. 85), Luxion and Johnston (AMR **2**, Rev. 21), Roderick and Phillipps (AMR **3**, Rev. 1948), Andrews (AMR **3**, Rev. 2278), and Heyman (AMR **4**, Rev. 2889); problems concerning plane frames by Baker and his collaborators (AMR **3**, Revs. 878, 2632; **4**, Rev. 2460), and problems on space frames by Heyman (AMR **3**, Rev. 2274). In addition to these papers on specific structural applications, there are a number of papers on general methods: Greenberg and Prager (AMR **3**, Rev. 2273), Neal and Symonds (AMR **3**, Revs. 2275, 2280; **4**, Revs. 1553, 1554, 1556, 1557, 2448), Yang, Beedle and Johnston (Rev. 4134 in this issue). Some of these papers are concerned with proportional increase of all loads up to the collapse of the structure, others with the case where a number of loads vary independently between given minimum and maximum values. As Horne (AMR **3**, Rev. 1928) pointed out, two types of failure are possible in the second case even though no single combination of the loads can produce the type of collapse encountered in the first case. As the loads continue to vary in a certain sequence between the given extreme values, cycles of plastic flow may occur in certain members, and these cycles will be repeated as often as the sequence of loads is repeated. Alternatively, intermittent plastic flow without reversal of the sense

of deformation may occur in certain members and continue as long as the load sequences are continued.

The problems of limit analysis have been discussed in geometric terms by Prager and Symonds (AMR 1, Rev. 1203; 4, Revs. 1127, 1558, 2025), and Rzhanitzin ("Some problems in the mechanics of systems which deform with time," Moscow, 1949, chap. I). A survey of the methods of limit analysis as applied to plane frames has been given by P. S. Symonds (AMR 4, Rev. 3204). Brief surveys of this field have been published by Baker (*The Structural Engineer* 27, p. 397, 1949; *The Civil Engineer in War* 3, p. 30, 1948). The extension of limit analysis to continuous media has been discussed by Feinberg (AMR 2, Rev. 486), Symonds (AMR 4, Rev. 2475), Drucker, Greenberg, and Prager (ASME Paper No. 51—A-3), Hodge and Prager (pp. 213–228 and 247–225) and Hill (*Phil. Mag.* (7) 42, 868–875, 1951).

5 STRUCTURAL STABILITY

From the practical point of view, the problem of column stability is concerned with the determination of the smallest axial load for which the column can assume a bent as well as a straight configuration. The classical treatment of column stability in the plastic range, however, is concerned with the more restricted problem of determining the smallest axial load under which the column can pass from the straight to a bent equilibrium configuration. As Shanley (AMR 1, Rev. 72) pointed out, the critical load is smaller for the first problem.

The majority of recent investigations of plastic buckling of plates and shells are based on total strain theories: Bijlaard (AMR 1, Revs. 613, 1206; 2, Revs. 453, 1378; 3, Rev. 1925), Kollbrunner and collaborators (AMR 1, Rev. 64; 2, Rev. 453; 3, Rev. 47), Ilyushin (AMR 1, Revs. 264, 804), Stowell (AMR 1, Revs. 805, 1331; 3, Revs. 458, 1079, 1467, 2621), Pride and Heimerl (AMR 2, Rev. 1124). Discussions based on incremental strain theories have been given by Kuntze (AMR 1, Rev. 262), Handelman and Prager (AMR 1, Rev. 1329), and Hopkins (AMR 4, Rev. 2427). Buckling experiments seem to speak in favor of the first type of analysis. This fact is frequently cited as an experimental confirmation of total strain theories. To this writer the idea of testing a stress-strain law by buckling experiments seems utterly fantastic; nobody would dream of determining, say, Young's modulus by a buckling test in the elastic range rather than a simple tension test. Direct experiments speak in favor of incremental strain theories, and there are strong theoretical objections against all total strain theories. In these circumstances, any acceptable analysis of structural stability in the plastic range must be based on an incremental strain theory. The failure of the afore-mentioned analyses of this kind to furnish results in agreement with experimental data is likely to be due to some or all of the following reasons: (1) The incremental strain theories used may not be sufficiently general (Batdorf, AMR 3, Rev. 1924; Drucker, AMR 3, Rev. 1926); (2) the effect of small initial imperfections may be sufficiently large to necessitate its inclusion into any satisfactory analysis (Lin, AMR 4, Rev. 1091; Pearson, AMR 4, Rev. 1097; Ciala, AMR 4, Revs. 1100, 1522, 2436; Duberg and Wilder, AMR 4, Rev. 1525); (3) creep may have an important effect (Marin, AMR 1, Rev. 257); and (4) it may be necessary to consider plastic buckling as a dynamic rather than a static problem (Hoff, AMR 3, Rev. 1892).

6 DYNAMIC PROBLEMS

The literature on dynamic problems of plasticity has been expanding rapidly ever since the war-time work in this field has become more readily accessible. In the period reviewed here, plane waves in homogeneous elastic-plastic and viscoelastic-plastic materials have been discussed by Sokolovsky (AMR 2, Rev. 571), White and LeVan Griffis (AMR 1, Revs. 220, 1450; 2, Rev. 839), Ogibalov and Loginova (AMR 3, Revs. 60, 259), Rakhmatoolin (AMR 2, Rev. 163), Lensky (AMR 3, Rev. 472), DeJuhasz (AMR 4, Rev. 94), Zverev (AMR 4, Rev. 1944), Malvern (AMR 4, Rev. 2372), and Lee and Wolf (ASME Paper No. 50—A-35 in *Mech. Engng. tech. dig.*, 36–37, Jan. 1951). Brief surveys of this field have been given by Thornton (AMR 1, Rev. 1093), and von Kármán and Duwez (AMR 4, Rev. 2481).

Plane waves in an inhomogeneous material were investigated by Rakhmatoolin (AMR 3, Rev 2202), spherical waves in homogeneous materials by Bakhshiyan (AMR 1, Rev. 1589), and Lunts (AMR 3, Rev. 692), and cylindrical shear waves by Rakhmatoolin (AMR 1, Rev. 976) and Sokolovsky (AMR 1, Rev. 1181; 4, Rev. 213). Transverse impact on beams has been studied by Duwez, Clark, and Bohnenblust (AMR 3, Rev. 2203), and impact on plates by Bakhshiyan (AMR 1, Rev. 836), and Kochetkov (AMR 4, Rev. 217).

7 MISCELLANEOUS PROBLEMS

(*a*) *Rigid-plastic solutions.* In the rigid-plastic solutions discussed in this paragraph, all elastic deformations are neglected. Nearly half of Sokolovsky's book is devoted to statically determinate problems in which plastic stress fields in two dimensions are constructed from given surface tractions. These statically determinate problems of plasticity have received an unjustified amount of attention ever since Hencky and Prandtl first discussed such problems in 1923. The ways in which a physically meaningful boundary-value problem can be set have been discussed by Lee (AMR 4, Rev. 2917); he showed that the determination of a plastic stress field from boundary conditions on surface tractions alone does not constitute a physically meaningful problem as a rule and that, in the large majority of meaningful problems, the determination of the stress field cannot be separated from that of the velocity field. In the following discussion solutions will be called "complete" or "incomplete" according to whether they take account of this interdependence of the stress and velocity fields or disregard it. In the period under review, work on incomplete solutions was concerned primarily with the determination of plastic stress fields in materials obeying general or new special types of yield conditions (Mandel, *C. R. Acad. Sci. Paris* 225, 1272–1273, 1947; von Mises, AMR 2, Rev. 1135; Neuber, AMR 3, Rev. 1475; Sauer, AMR 3, Rev. 2647; Hodge, AMR 4, Rev. 1148; Geiringer, AMR 4, Rev. 3548), and the discussion of discontinuous stress fields (Prager, AMR 2, Rev. 186; Carrier and Winzer, AMR 2, Rev. 472; 3, Rev. 1929; Hodge, AMR 4, Rev. 1566). These discontinuous stress fields have been used successfully in limit analysis (see, for instance, Prager and Hodge, p. 215). Complete solutions of technologically important problems of plastic flow have been presented by Hill, Lee, and Tupper (AMR 1, Revs. 102, 847, 1647; 2, Rev. 650; 3, Revs. 473, 1927; 4, Rev. 3224; ASME Paper No. 51—A-4) and Lee and Shaffer (ASME papers Nos. 51—A-5 and 51—A-8).

(*b*) *Elastic-plastic solutions.* General analytical methods of obtaining elastic-plastic solutions to problems, where the shape of the elastic-plastic interface is not known beforehand, have not yet been developed, but a few particular problems of this kind have been attacked successfully by Galin (AMR 1, Rev. 1353; 3, Rev. 1097; 4, Rev. 218) and Parasyuk (AMR 2, Rev. 470). Other problems of this type have been solved by relaxation methods (Shaw and Eddy, AMR 3, Rev. 1652; Southwell and Allen, AMR 3, Rev. 2317; 4, Rev. 203; Jacobs, AMR 4, Revs. 669, 670). Elastic-plastic problems become more tractable when the elastic-plastic interface has a known shape and a position depending on one parameter only. This is the case for spheres or circular tubes under internal or external pressure (Hill, Lee, and Tupper, AMR 1, Rev. 282; MacGregor, Coffin, and Fisher, AMR

1, Rev. 979; Taylor, AMR 2, Rev. 41; Nadai, AMR 2, Rev. 1389; Hodge and White, AMR 4, Rev. 1568; Mii, AMR 4, Rev. 2483; Allen and Sopwith, AMR 4, Rev. 3181) or for beams in combined bending and tension (Swift, AMR 2, Rev. 621). Similar simplifications arise in the case of thin-walled sections (Mandel, *Ann. Ponts Chauss.* 116, 1–33, 1946; Handelman, AMR 4, Rev. 1518; Hill and Siebel, *Phil. Mag.* (7) 42, 722–733, 1951). An approximate treatment of secondary stresses in bent beams has been given by Craggs, *Quart. J. Mech. appl. Math.* 4, 241–247, 1951. Swida (AMR 3, Rev. 471) has discussed the bending of bars with initial curvature. Lubahn and Sachs (AMR 4, Rev. 132) have treated the finite flexure of a beam, taking account of the charges in cross section and the corresponding displacement of the neutral axis.

(*c*) *Solutions based on total strain laws.* In the period under review, total strain laws have been widely criticized on experimental as well as theoretical grounds (see Sec. 2 of this survey). The rather restrictive conditions under which the predictions of a total strain law agree completely with those of a suitable incremental strain law are well known. In spite of this, total strain laws continue to be applied to problems where these conditions are not rigorously fulfilled. Comparative studies of particular solutions based on the two types of stress-strain laws have been made by Hodge and White, Hill and Siebel, and Allen and Sopwith in the papers mentioned under (*b*) above. The results of these authors would seem to indicate that some latitude is permissible in the application of the conditions for agreement between total and incremental strain laws. It is with this in mind that the following work based on total strain laws is mentioned: Sokolovsky (AMR 1, Rev. 798; 3, Rev. 2316; 4, Revs. 201, 2028), MacGregor, Coffin, and Fisher (AMR 1, Rev. 977), Gleyzal (AMR 2, Rev. 26), Kachanov (AMR 2, Rev. 45), Rabotnov (AMR 2, Rev. 46), Phillips (AMR 2, Rev. 1137), Ilyushin (AMR 2, Rev. 1503), Panferov (AMR 3, Rev. 691), Moufang (AMR 3, Rev. 693), Shevchenko (AMR 3, Rev. 1095), Shapiro (AMR 4, Rev. 202), Savin and Parasyuk (AMR 4, Rev. 680), Kostyuk (AMR 4, Rev. 1519), Wu (AMR 4, Rev. 2027), Swida (AMR 4, Rev. 2871), Mii (AMR 4, Rev. 3231). The solutions presented in these papers should be used with caution until more is known about the conditions for *practical* agreement between solutions based on the two types of stress-strain laws.

Communications

Concerning Rev. 3012 (July 1951): Deissler, R. G., Analytical investigation of turbulent flow in smooth tubes with heat transfer with variable fluid properties for Prandtl number of 1.

It is stated in the review that the analysis utilizes the mutually contradictory assumptions that shear stress and static pressure are constant across the tube. It is also stated that a refinement of the usual theory is computed while retaining the crude approximation of constant shear stress across the tube. In connection with these statements, the reviewer failed to point out that the use of the above assumptions did not appreciably affect the results of the analysis. The variation of shear stress across the tube has but a slight effect on the computed velocity distributions even though the shear stress actually goes to zero at the tube axis. An investigation of the effect of variable shear stress with heat transfer is being given in a later paper.

By a slight change in the wording of the assumption concerning the shear stress, it could be made consistent with the assumption of constant static pressure. It could have been stated as: "The variation of shear stress across the tube has a negligible effect on the velocity distribution."

Robert G. Deissler, USA

Concerning Rev. 1541 (April 1951): Falkenheiner, H., Systematic calculation of elastic characteristics of statically indeterminate systems.

In a letter to the editor, the author, H. Falkenheiner, points out that the value of his method consists not only in the solution but also in the systematic setting-up of the required elasticity equations, by means of the integration matrix. He points out that this method is not identical with Beyer's. This was not maintained by the reviewer who merely quoted Beyer's work as an example of earlier matrix solutions and acknowledged Falkenheiner's "special approaches." The author notes that the positive, definite matrixes he obtains are also convenient for vibration calculations.

George Winter, USA

Concerning Rev. 785 (February 1951): Magyar, F., On the derivation of Crocco's vortex theorem.

The reviewer apologizes for his failure to check the author's quotation of the work of Oswatitsch. As pointed out to the reviewer by Dr. Sune B. Berndt, the author's Eq. (1) is not to be found in Oswatitsch's paper, where the equations and the analysis are perfectly correct. The "divergence of opinions" thus appears to be created by the author.

C. Truesdell, USA

Concerning Rev. 2956 (July 1951): Campus, F., Influence of length on transverse shrinkage of butt welds.

Author does not agree with reviewer about the facts. In the range from 1 to 200 cm covered by the research, specific shrinkage changes steadily with length of butt weld. The maximum scatter is ±31%; the mean value is much less. Mathematical criticism of the reviewer is not appropriate to author's definite purpose. Logarithmic formula is an empirical expression and interpretation of experimental results in the mentioned range. Infinite length is without significance. Formula is harmless for extrapolation till maximum practical length of butt welds, perhaps on the safe side. Reviewer made the mistake of reading the length of the tanker weld as 2 m instead of 6 m.

F. Campus, Belgium

Theoretical and Experimental Methods

(See also Revs. 4091, 4101, 4120, 4165, 4223, 4224, 4325)

4042. Langhaar, H. L., Dimensional analysis and theory of models, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1951, xi + 166 pp. $4.

This well-written book begins with a review and familiarization of fundamental dimensions. Chapter II introduces the Buckingham theory with examples of application. Chapter III contains a method of determining the number of dimensionless products and the use of determinants in establishing a system of solutions. Chapter IV is devoted entirely to rigorous mathematics leading to the introduction of the π function and the Buckingham theory. Introduction to, and some applications of model testing with related dimensional interpretations are contained in Chapter V. The remainder of the book is devoted to application of dimensional analysis and model testing to various physical fields.

Reviewer believes this book to be the foundation for a more comprehensive understanding of dimensional analysis to model application.

Ernest G. Stout, USA

4043. Buslik, D., Mixing and sampling with special reference to multi-sized granular material, *Amer. Soc. Test. Mat. Bull.* no. 165, 66–73, Apr. 1950.

Author derives a formula for the standard deviation of the percentage of particles of a given size in random samples (by weight or volume) from a granular material with known size distribution. It is shown how this formula is applicable to (a) the problem of estimating the unknown proportion of particles of any size (or size range), (b) the study of the mixing of two or more uniformly sized materials, and (c) the study of the arrangement of particles in multisized aggregates. Discussion of the mixing operation from a statistical point of view is presented. Formula is applied to artificially constructed mixtures (random numbers) and to empirical data.

Nils G. Blomqvist, Sweden

4044. Kassander, A. R., Jr., and Stebbins, D. W., The application of electrical counting to the compilation of frequency distributions and correlation tables, *Trans. Amer. geophys. Un.* **32**, 3, 341–346, June 1951.

An electrical-mechanical tabulator is described which presents statistical data in array form immediately on the conclusion of a test, thereby eliminating the time-consuming process of measuring and transferring data from tape records. The basis of operation is to cause differential relays to direct a sampling pulse to appropriate counters representing different class intervals of the variable being examined. Thus, the number of counts in each counter is proportional to the length of time that the variable was in a particular class interval. In its present form, the instrument is capable of presenting a nine-class frequency distribution for one variable, a twenty-five cell correlation diagram for two variables, or six tetrachoric correlation diagrams for four variables.

From authors' summary

4045. Fröberg, C.-E., On the solution of ordinary differential equations with digital computing machines, *Acta Polyt.* no. 79, 17 pp., 1950.

Part of preparing a problem for an automatic electronic computing instrument (e.g., SEAC, ENIAC, ORDVAC, etc.) is reducing the numerical solution to a sequence of simple arithmetic operations. This is called coding the problem. Author gives the flow diagram and final coding for the numerical integration of a system of first-order first-degree differential equations by the Runge-Kutta method [cf. AMR **4**, Rev. 973]. The one-address code proposed by von Neumann and Goldstine is used. The coding for the special cases $y_x = f(x)$ and $y_x = f(x, y)$ is also given.

C. L. Perry, USA

4046. Kaplan, C., On a solution of the nonlinear differential equation for transonic flow past a wave-shaped wall, *Nat. adv. Comm. Aero. tech. Note* 2383, 35 pp., June 1951.

Paper treats problem of compressible potential flow past an infinite sinusoidal wall of small amplitude. A formal series solution is obtained by expanding equation for potential and boundary conditions in powers of thickness, with basic subsonic free stream. When transonic approximation is made in this series, it is shown to be identical with a series, expressed in terms of similarity parameter $k = (\gamma + 1)\pi t(1 - M^2_\infty)^{-3/2}$, t = thickness ratio, obtained directly from transonic equation for the potential by using an iteration method. For the series, critical Mach number is estimated to be $k = 0.84$. An attempt to determine radius of convergence of series in k results only in necessary condition $k < 4/3$. Thus no conclusion can be drawn about validity of expansion method when local supersonic zones are present.

Julian D. Cole, USA

4047. Johansen, K. W., On difference equations (in Danish), *Frandsen Anniv. Vol. Lab. Bygn. Tekn. Medd.* no. 1, 64–70, 1950.

Paper deals with evaluation of the stress function, its integral, and partial derivatives from Poisson's equation. Method used employs finite differences in connection with Taylor's series. Improved accuracy is obtained without reducing the interval length. Method is illustrated by two numerical examples. For a rectangular cross section with sides $2a$ and a and an interval length of $\frac{1}{2} \cdot a$, the polar moment of inertia is determined with $1^1/_2\%$ accuracy by method.

T. A. Mortensen, USA

4048. Allen, D. N. de G., and Dennis, S. C. R., The application of relaxation methods to the solution of differential equations in three dimensions. I. Boundary value potential problems, *Quart. J. Mech. appl. Math.* **4**, part 2, 199–208, June 1951.

A simple method of using relaxation method in three dimensions is developed. Isometric projection of three-dimensional lattice enables the work to be carried out on a single sheet of paper. Formulas have been given for the case of irregular star in three dimensions. Illustrative examples are given.

G. V. R. Rao, India

4049. Luke, Y. L., and Ufford, D., Concerning a definite integral, *J. aero. Sci.* **18**, 6, 429–430, June 1951.

In the aerodynamic theory of oscillating wings of finite span, the definite integral arises: $F(iz) = {}_0\int^\infty e^{-it}(z + t - (z^2 + t^2)^{1/2}dt/zt = {}_0\int^\infty e^{-iz \sinh \theta}(1 + \sinh \theta - \cosh \theta) \cosh \theta \, d\theta / \sinh \theta$. Authors show that it can be expressed completely in terms of tabulated functions.

Ed.

4050. Shenker, H., Lauritzen, J. I., Jr., and Corruccini, R. J., Reference tables for thermocouples, *Nat. Bur. Stands. Circ.* 508, 71 pp., May 1951.

Expanded reference tables for common commercial thermocouples are given. The tables incorporate recent changes in electrical units and temperature scale. From authors' summary

4051. Van Dyke, M. D., Young, G. B. W., and Siska, Ch., Proper use of the M.I.T. tables for supersonic flow past inclined cones, *J. aero. Sci.* **18**, 5, 355–356, May 1951.

One of the reports discussed in the paper is reviewed in AMR **3**, Rev. 935.

4052. Boulanger, G. R., General theory of superposed plane nomograms [Théorie générale des abaques à plans superposés], Paris, Gauthier-Villars, 1949, 117 pp., 30 figs., 4 tables. Fr. frs. 1200.

Maurice d'Ocagne, founder of the science of nomography, foresaw (1893) a large field of sliding charts. A well-known slide rule is an example of such a chart. W. Margoulis (1931) applied this principle to superposed nomograms, one of them drawn on transparent paper. Author contributes to the general theory of compound nomograms, amplifying their application to the unlimited number of variables involved. An extensive terminology of nomograms is defined; there are to be distinguished: (1) Complete scales—consisting of lines, degenerated—consisting of points, and derived—with some constants; (2) charts with tangential or pointed contact; (3) these on independent or conjugated planes; (4) simple and compound; (5) for total or partial solutions; (6) common or specials. A specific schematic representation of the construction of charts is introduced. Author details the charts with tangential contact, on independent planes as well as on conjugated planes.

This dissertation is an excellent theoretical essay of the gen-

eral nomography as a base for its further development. Engineers, mainly interested in the application of nomography, will appreciate having an additional practical treatise on the superposed sliding charts. Steponas Kolupaila, USA

4053. Hort-Thoma, The differential equations of technology and physics [Die Differential-Gleichungen der Technik und Physik] 5th ed., Leipzig, Johann Ambrosius Barth, 1950, xii + 576 pp. DM 46.80.

This 5th edition corrects some errors and misprints in the fourth and contains some new material, principally a discussion of Schroedinger's equation and its application to the harmonic oscillator and the hydrogen atom. Though the volume is perhaps well-known, its refreshing style and manner of approach to problems in applied mechanics deserve a few remarks. It is a valuable aid to the student of applied mathematics. The introduction of almost every topic is accompanied by an illustration from physics or technology. For example, the special functions of mathematics including those of Bessel, Legendre, Loguerre, etc., are introduced by showing how they arise naturally from a mathematical description of certain physical phenomena. The differential equations of potential theory, elasticity theory, hydrodynamics, and electrodynamics are but a partial list of topics covered. Two short chapters are devoted to variational calculus and to a discussion of linear integral equations. A complete table of contents is available. An interesting feature is an appendix listing the differential and partial differential equations with the text page number where they occur. Equations known by a name associated with a physical phenomenon are so labeled.

Y. Luke, USA

4054. Zavrotsky, A., Tables of solutions of equations of fifth degree (in Spanish), *Bol. Acad. Cienc. Fis. Mat. Nat. Venezuela*, **13**, 41, 51–93, May–Aug. 1950.

Tables contain real roots of the equation $x^5 = px^3 + qx^2 + rx + 1$ for integers p, q, r between +10 and −10. Ed.

4055. Dwyer, P. S., Linear computations, New York, John Wiley & Sons, Inc.; London, Chapman and Hall, Ltd.; 1951, xi + 344 pp. $6.50.

This volume provides the applied worker with methods and techniques for the solution of simultaneous linear equations, the evaluation of determinants, the computation of the inverse and adjoint matrix, and the solutions of the characteristic-value problem. Other topics are computation with approximate numbers, computational design, errors of linear computations, and applications to statistics and nonlinear problems. The book is rich in numerical examples and exercises. Emphasis is placed on those methods readily adaptable to modern desk calculators. The book is written in a simple, easy-to-understand fashion, and only an elementary knowledge of algebra is required.

Y. Luke, USA

4056. Ascoli, G., Remarks on some stability questions. I (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **9**, 129–134, 1950.

Author discusses some questions connected with the boundedness of solutions of the vector-matrix equation $dy/dt = (A + B(t))y$, where $B(t)$ is small in some sense as $t \to \infty$, and their relation to known results. R. Bellman, USA

4057. Ascoli, G., Remarks on some stability questions. II (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **9**, 210–213, 1950.

Author shows by means of ingenious transformations that all the solutions of $u'' + (1 + kt^{-1} \sin at)u = 0$ are bounded for $a \neq 2$, while this result is not true for $a = 2$. A separate discussion is necessary in this case since it eludes the known stability criteria which require absolute integrability or bounded variation, due to Ascoli. R. Bellman, USA

4058. Chalmette, M., English-French vocabulary of aeronautical terms, *Aircr. Engng.* **23**, 268, 179–180, June 1951.

4059. Petschacher, M., Tables of hypergeometric functions (in Italian), *R. C. Mat. appl.* (5), **9**, no. 3–4, 1950 = *Cons. Naz. Richer.* no. 297, 32 pp., 1951.

For applications in fluid dynamics, $F(a,b;c;x)$ and $x^{(c-1)/2}F(a,b;c;x)/\Gamma(c)$ are tabulated, where $a = \mu/2 + 1/2\,(k - 1) + [(k^2 - 1)\mu^2 + 1]^{1/2}/2(k - 1)$; $b = \mu/2 + 1/2(k - 1) - [(k^2 - 1)\mu^2 + 1]^{1/2}/2\,(k - 1)$; $c = 1 + \mu$.

The tabulations, usually to 6 significant figures, are for μ = 0.125(.125) 0.875, 1.25(.25) 2(.5) 7 (1) 10, k = 1.4, x = 0(.02) 1.

Peter Chiarulli, USA

4060. Milne, W. G., Bibliography of seismology, *Publ. Dom. Observ.* **14**, 8, 167–185, 1951.

4061. Fiock, E. F., and Halpern, C., Bibliography of books and published reports on gas turbines, jet propulsion and rocket power plants, *Nat. Bur. Stands. Circ.* 509, 64 pp., June 1951.

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 4077, 4083)

4062. Schmid, W., On the coupler curve of the crosshead mechanism (in German), *Z. angew. Math. Mech.* **30**, 11/12, 388–390, Nov./Dec. 1950.

A study is made of the geometrical properties of the fourth-degree curve traced by a point carried by the connecting rod of a crosshead mechanism. For example, it is shown that the two double points of the curve lie on a line through the center of rotation of the crank. This line has the same angle with the normal to the crosshead track as the angle between the two lines joining the carried point to the pivots of the connecting rod.

M. Goldberg, USA

4063. Meyer zur Cappellen, W., On the three-bar curves of the twin mechanism (in German), *Z. angew. Math. Phys.* 2, 3, 189–207, May 1951.

Certain three-bar curves of a twin mechanism may be used as wing profiles [Piercy, Piper, Preston, *Phil. Mag.* (7) **24**, 425–444, 1937]. Author discusses geometrical properties of the curve. It is generated by a point P of the plane of the moving bar AB of a three-bar mechanism A_0B_0BA with the fixed points A_0, B_0 and such that $A_0B_0 = AB = c$, $A_0A = B_0B = a < c$ (antiparallelogram mechanism). The curve is bicircular of degree 4. Polar equation of the curve. If P is the middle point of AB, the curve looks like a lemniscate. Author shows that the arbitrary curve may be found as a deformation of this one. Construction of the radius of curvature in some particular points. Circumscribed and inscribed circle of the curve. The paper contains some good graphs of various types of the curve in question.

O. Bottema, Holland

4064. Bottema, O., On Grübler's formulae for mechanisms, *Appl. sci. Res.* A2, **2**, 162–164, 1950.

This is an elementary derivation of Grübler's formulas for the

least number of degrees of freedom x of a linkage of n links. If s_i are the numbers of the couples of multiplicity q_i ($q = 2$ for a plane hinge, etc.) and p is the number of degrees of freedom of the free rigid body (for plane linkages, $p = 3$; for spatial linkages, $p = 6$), the general formula is $x = p(n - 1) - \Sigma q_i s_i$.

Michael Goldberg, USA

4065. **Artobolevskiĭ, I. I., On some forms of the equations of motion of a machine aggregate** (in Russian), *Doklady Akad. Nauk SSSR* (*N.S.*) **77**, 6, 977–979, Apr. 1951.

The dynamical equations of motion of a mechanism are expressible by $M = d/dt(\partial T/\partial\omega) - \partial T/\partial\phi$, where $T = I\omega^2/2$, ϕ is the position angle, ω is the angular velocity, and M (the resultant turning moment) and I (the polar moment of inertia) are functions of ϕ, ω, and the time t. Various known special cases are derived in which some of the quantities are independent of one or more of the variables.

Michael Goldberg, USA

4066. **Reuschel, A., Vehicle motion in a line** (in German), *Öst. Ing.-Arch.* **4**, 3–4, 193–215, 1950.

For driving in line, the traffic regulations should specify that each car always have a distance from the preceding car of so many meters as the instantaneous speed expressed in km/h plus the distance prescribed for the cars being at rest. Starting with this distance regulation and the motion of the leading car being known, both an analytical and a graphical method are indicated on how to find out the variation of the motion of all following cars. Finally, the admissibility of the above-mentioned traffic regulations is examined in the special case of the leading car, moving with constant speed, being stopped suddenly.

From author's summary

⊗4067. **Timoshenko, S., and Young, D. H., Engineering mechanics, 3d ed.,** New York, McGraw-Hill Book Co., Inc., 1951, xiv + 517 pp., 736 figs. $5.50.

The make-up of the third edition is similar to that of the second edition, but many details of the text have been improved by the revision. There are 195 examples and 650 problems divided about equally between the statics and dynamics parts, while the second edition had 567 problems, some of which were in the nature of illustrative examples. The main change in the third edition, therefore, is the addition of a large number of illustrative examples and many excellent problems.

Part 1, dealing with statics, starts with the introduction of force systems and proceeds logically through concurrent forces in a plane, parallel forces in a plane, general case of forces in a plane, forces in space, and ends with a chapter on virtual displacement.

Part 2, dealing with dynamics, starts with the principles of dynamics and proceeds through rectilinear and curvilinear translation, rotation about an axis, plane motion, and relative motion. There are also two appendixes on moments of inertia of plane figures and solid bodies.

In the statics part, the inclusion of a chapter on virtual displacements with its many excellent examples and problems is particularly gratifying; in the dynamics part, d'Alembert's principle is used extensively and to good advantage.

Another feature of the book is that the sections on momentum, angular momentum, and energy are scattered all through the dynamics part of the book whenever it is appropriate to discuss these topics, rather than having them collected in separate chapters.

In the opinion of the reviewer, the book is well balanced as to contents, and its clear, rigorous style puts it in the forefront among undergraduate textbooks on engineering mechanics. Reviewer's only criticism is that free-body diagrams could have been used more freely in setting up the equations of equilibrium.

N. O. Myklestad, USA

Courtesy of Journal of Applied Mechanics

4068. **Quinn, B. E., Energy method for determining dynamic characteristics of mechanisms,** *J. appl. Mech.* **16**, 3, 283–288, Sept. 1949.

In many mechanisms the position of all parts is completely defined by a single variable. In a common motor, e.g., the position of all parts is determined by the angle by which the crankshaft has turned. For such mechanisms the following theorem is advanced and is evidently true: "The percentage of the total kinetic energy which the link of a mechanism contains will remain the same in any given position regardless of the speed."

The method proposed in paper consists, first, in drawing curves showing the percentage of the total energy contained in the link in question for different values of the variable. From diagrams of the driving forces and the friction losses vs. the variable mentioned above, it is possible to obtain the gains and the losses in the total kinetic energy. By aid of the first-mentioned diagram, a numerical or graphical integration gives the connection between position and velocity for the link in question. From that result one obtains the acceleration and the velocity-time diagram by usual methods.

To exemplify the method, paper contains the complete solution of a very simple problem.

O. H. Faxén, Sweden

4069. **Colombo, G., Remarks on the stability of merostatic motions of a gyroscope and application to an important case** (in Italian), *R. C. Semin. mat. Univ. Padova* **20**, part 1, 59–77, 1951.

The general expression for a moment vector acting on a gyroscope is derived, which produces a merostatic motion, i.e., a regular precession or a constant rotation about an axis different from the gyro axis. Discussion of this expression yields the general stability criterion: If this moment vector is isolated the merostatic motion is stable. If, in the infinitesimal vicinity of the moment vector other vectors exist which also cause merostatic motions, the motion is unstable.

For motions representable by quadratures, stability with respect to the Eulerian coordinate θ is secured if $(\sin\theta\dot{\theta})^2$ has a maximum; a minimum of this function indicates instability. Result is applied to two examples.

Gerhard W. Braun, USA

⊗4070. **Lyapunov, A. M., General problem of the stability of motion [Obshchaya zadacha ob ustoĭchivosti dvizheniya]** (in Russian), Moscow, Leningrad, Gosud. Izdat. Tekhn.-Teor. Lit., 1950, 471 pp., 1 plate.

In addition to the paper of the title [Kharkow, 1892 = *Ann. Fac. Sci. Univ. Toulouse* (2) **9**, 203–474, 1907] this volume contains the following papers: *Comm. Soc. Math. Kharkow* (2) **3**, 265–272, 1893; *Mat. Sbornik* **17**, 253–333, 1893; *J. Math. pures appl.* (5) **3**, 81–94, 1897.

Courtesy of Mathematical Reviews

4071. **Colombo, G., On equilibrium configurations of a flexible and inextensible, developable sail** (in Italian), *R. C. Semin. mat. Univ. Padova* **20**, part 1, 153–166, 1951.

Referring to earlier Italian papers of E. Laura, E. Beltrami, C. Tolotti and others, the equilibrium of a flexible and inextensible surface in space is investigated, similar to a sail which is loaded with forces in the following manner: (a) weight, (b) wind, or (c) constant pressure in the direction of the normal to the surface. The conditions for the existence of *regular* forms for the equilibrium configuration are set up; this means that the functions $x = x(u, v)$, $y = y(u, v)$, and $z = z(u, v)$ have, in the con-

sidered range, derivatives of the first and second order. The considered forms of the surfaces are triangles and rectangles, also other forms with nonlinear edges. This is a problem of differential geometry and requires for its solution the methods of the applicability of surfaces to a plane.

Theodor Pöschl, Germany

Gyroscopics, Governors, Servos

(See also Revs. 4069, 4260)

4072. Lur'e, A. I., On strictly unstable regulated systems (in Russian), *Prikl. Mat. Mekh.* **15**, 2, 251–254, Mar.-Apr. 1951.

The equations of motion of a servomechanism are put in the canonical form $\dot{z}_t = \mu_t z_t + \varphi(\sigma)$, $\sigma = \sum \nu_t z_t$, $t = 1, 2, \ldots, n + 1$, and a Lyapunov function is constructed for the system.

Courtesy of Mathematical Reviews J. G. Wendel, USA

4073. Lur'e, A. I., On the problem of the stability of regulated systems (in Russian), *Prikl. Mat. Mekh.* **15**, 1, 67–74, Jan.-Feb. 1951.

Paper represents a continuation of two previous papers [title source, **9**, 1945 and **12**, 1948; see AMR **3**, Rev. 829]. Author states that in the first paper he shows that the establishment of sufficient conditions of stability of certain control systems reduces to the investigation of roots of n quadratic equations in the general case. In present paper he takes the results obtained in the second paper and carries a rather long series of formal transformations of quadratic forms which cannot be abstracted here. The end of the paper is devoted to the discussion for $n = 2, 3$, and 4. For $n = 2$, it is shown that the results agree with the conclusions of Letow [op. cit. **12**, 1948]; for $n = 3$, the sufficient condition requires that a certain quartic equation must have at least one pair of real roots. The author mentions that for $n = 4$ the calculations are too complicated to be of interest, but shows that for certain conditions this case reduces to the previous one.

N. Minorsky, France

4074. Blanot, H. E., Carrier compensation for servomechanisms, *J. Franklin Inst.* **250**, 5, 6; 391–407, 525–542; Nov., Dec. 1950.

Paper deals with methods for improving carrier-type servomechanism performance by "compensating networks." Subject is of profound engineering significance because majority of servomechanisms in practice are of carrier type (those using synchro data, for example). On the other hand, most texts deal primarily with the simpler problem of compensating noncarrier types. Only a few random papers deal with carrier-servomechanism compensation. Paper is a critical summary of previous contributions; author proposes no new methods. Basic requirements for carrier-compensation schemes are described. Author classifies schemes as follows: (1) Networks operating directly on modulated carrier (electrical); (2) demodulation, compensation, remodulation (electrical and/or mechanical).

First classification involves passive networks. It is shown that only phase-lead compensation is feasible with present techniques and that desired compensating transfer functions may be obtained only approximately. Such networks may be of two possible types: (a) *RLC* networks, following method proposed principally by A. C. Hall, can obtain approximately any desired transfer function. Necessity of using inductances is cited as a disadvantage. (b) *RC* networks, following methods proposed principally by Sobczyk, are more convenient and avoid inductances. They are more difficult to design and adjust and are more limited in types of functions obtainable. Both general methods suffer from carrier-frequency shifts. Method proposed by Notthoff for eliminating this disadvantage is discussed.

The second classification involving demodulation of carrier signal is discussed. Methods proposed by MacDonald involving electromechanical demodulation and mechanical compensating networks are explained. These methods do not suffer from carrier-frequency shifts but are relatively complex and expensive.

The relative advantages of the various methods are cited. Reviewer feels that the paper presents a valuable summary of methods proposed to handle this general problem. Several papers have appeared on this subject since this one was written. Among them are papers by G. M. Attura and G. A. Bjornson, presented before the Committee on Feedback Control Systems of the American Institute of Electrical Engineers in January, 1951. These papers will probably appear later in that Institute's *Transactions*.

Ralph J. Kochenburger, USA

Vibrations, Balancing

(See also Revs. 4321, 4323, 4326)

4075. Burgreen, D., Free vibrations of a pin-ended column with constant distance between pin ends, *J. appl. Mech.* 18, 2, 135–139, June 1951.

Author considers free transverse vibrations of pin-ended column with pinned points fixed in space. His differential equation yields to product solution provided even-order derivatives of modal function differ from that function by a constant only.

Accordingly, he assumes initial sine curvature and obtains nonlinear time function equation having elliptic integrals for solutions. Three forms of solution appear, depending on frequency to amplitude ratio. The agreement in frequency and wave shape between theoretical and experimental values is very satisfactory.

R. M. Rosenberg, USA

4076. Okumura, A. and Yokoe, K., On free lateral vibrations of a hooked cantilever, *Memo. Fac. Sci. Engng. Waseda Univ. Tokyo*, no. 14, 30–31, 1950.

Free lateral vibrations of a hooked uniform cantilever are calculated and the results are verified by some model experiments.

Jaroslav Kozesnik, Czechoslovakia

4077. Colombo, G., On the stability of equilibrium configurations of a flexible and inextensible shell (in Italian), *R. C. Semin. mat. Univ. Padova* **19**, 214–230, 1950.

The equilibrium of an inextensible but flexible shell is absolutely stable if the two principal stresses σ_1, σ_2 are tensions (Condition: $\sigma_x, \sigma_y > 0$ and $\sigma_1\sigma_2 \equiv \sigma_x\sigma_y - \tau^2 > 0$). But this condition is too strict: considering small vibrations around the position of equilibrium one can see that the eigenvalues are purely imaginary (the vibrations stable), if the integral over a certain quadratic expression consisting of the derivatives of the displacements with the three stresses as coefficients is positive. For the special case of a cylindrical shell (e.g., triangular gutter), author proves that this latter condition may be realized (stability), the condition of positive stresses (absolute stability) being violated in a certain part of the shell.

K. Marguerre, Germany

4078. Mindlin, R. D., Thickness-shear and flexural vibrations of crystal plates, *J. appl. Phys.* **22**, 3, 316–323, Mar. 1951.

The theory of flexural motions of elastic plates, including the effects of rotatory inertia and shear, is extended to crystal plates. The equations are solved approximately for the case of rectangular plates excited by thickness-shear deformation parallel to one edge. Results of computations of resonant frequencies of rec-

tangular, so-called AT-cut, quartz plates are shown and compared with experimental data. Simple algebraic formulas are obtained relating frequency, dimensions, and crystal properties for resonances of special interest in design.

From author's summary by K. Klotter, USA

4079. Barton, M. V., Vibration of rectangular and skew cantilever plates, *J. appl. Mech.* **18**, 2, 129–134, June 1951.

The Ritz method is used to find the natural frequencies and normal modes of these types of plates. The characteristic functions known for beams are used for approximate plate deflections. Numerical evaluations are made for the first three symmetrical modes and first two antisymmetrical modes corresponding to several rectangular shapes and rhombus plates of skew angle 15°, 30°, 45°. Experimental work indicates that the calculated natural frequencies and nodal lines are correct for the smaller skew angles. It is concluded that the use of beam functions is not satisfactory for determining the two lowest frequencies for skew angles exceeding about 30°. R. E. Roberson, USA

4080. Bechmann, R., Contour modes of square plates excited piezoelectrically and determination of elastic and piezoelectric coefficients, *Proc. phys. Soc. Lond. Sec. B*, **64**, part 4, 376 B, 323–337, Apr. 1951.

Paper begins with a summary of recently derived approximate formulas for various modes of vibration of square plates and thin bars, followed by the derivation of the parameters of the equivalent electric circuits of such plates and bars when excited piezoelectrically.

Formulas were tested by measuring the resonant frequency and dynamic capacitance of a number of specimens chosen so as to give independent values of one or more of the elastic and piezoelectric coefficients; the extreme values for a given material differed by about 4%, which is not unreasonable in view of the approximations made. Materials used were sodium chlorate, sodium bromate, quartz, ammonium dihydrogen phosphate, and ethylene diamine tartrate. New values of the elastic and piezoelectric coefficients of these materials are given and compared with previous results. R. M. Davies, Wales

4081. Bruce, V. G., A graphical method for solving vibration problems of a single degree of freedom, *Bull. seism. Soc. Amer.* 41, 2, 101–108, Apr. 1951.

Author considers in detail a nonlinear single-degree-of-freedom system in which the restoring force vs. displacement curve is linear with negative slope in the region of the origin, and is linear with positive slope outside this region. It is noted that trajectories in the phase plane are hyperbolic or circular arcs according as the slope is negative or positive. The problem is considered in relation to a seismograph-type "boom pendulum."

R. W. Traill-Nash, Australia

4082. Vogel, Th., Topological methods of discussion of nonlinear vibration problems (in French), *Ann. Télécommun.* 6, 1, 1–9, Jan. 1951.

Paper presents the general topological theory, originated by Henri Poincaré from 1881 to 1885 and further developed by I. Bendixson in 1901 and H. Dulac in 1904 and later, as it applies to nonlinear oscillations. Author also reviews work of Russian contributors A. Andronov, L. Mandelstam, and A. Witt, as well as that of N. Minorsky and J. J. Stoker in the United States. Conditions on the roots of the equations which lead to nodes, necks, centers, etc., of the functions plotted in the phase plane are given.

Applications of the theory to physical problems occur in the second half of the paper where conditions leading to stable and unstable limit cycles are shown. Among others, the familiar problem of the pendulum with viscous friction having large oscillations is discussed. Examples include second-order first-degree differential equations in dy/dx and nonlinear in y.

Will J. Worley, USA

4083. Zhevakin, S. A., On the finding of limit cycles in systems near to certain nonlinear ones (in Russian), *Prikl. Mat. Mekh.* **15**, 2, 237–244, Mar.-Apr. 1951.

Poincaré's method of small parameter is applied to the calculation of a self-excited vibrating system, defined by n differential equations of the first order, near to another analogous nonlinear dynamical system whose $n - 1$ one-valued analytical integrals are known. Contrary to Hamilton's equations, n can also be odd.

The case of $n = 3$ is treated thoroughly, being of interest for the theory of pulsations of a cepheid; the method of investigation and the results can be immediately transferred to the general case of n equations.

From author's summary by Dragoš Radenković, Yugoslavia

4084. Lewis, R. C., The design and characteristics of a high sensitivity direct current operated accelerometer, *J. acoust. Soc. Amer.* **22**, 3, 357–361, May 1950.

This accelerometer uses the RCA 5734 tube, with an added mass for inertia, as the deflection-sensitive element. A diaphragm at the tube end, acting as spring element, supports the anode which projects through it, and this anode extension carries the mass. Either lateral or rotational accelerations produce relative deflections between the mass and the tube mounting. The natural frequency of the instrument is determined by the mass and the stiffness of the diaphragm; sensitivity is determined by the moment of the mass about the diaphragm center. The pickup alone, in case, weighs only two ounces. Anode deflections can be limited to ±0.005 in. for translation, or to $\pm 1/2°$ for rotation. By using two accelerometers, either translation or rotation readings can be obtained. Damping and phase shift are discussed. Ruggedness and dependability are claimed, together with good dynamic sensitivity. Eastman Smith, USA

4085. Weber, M., A new vibration-measuring apparatus and its application (in German), *Schweiz. Arch.* **17**, 5, 129–139, May 1951.

Theory and actual construction of vibrometers are dealt with. Author re-examines the equation of motion of a pendulum and constructs a polar diagram showing the relation between amplitude and phase in case of a forced vibration. Sources of errors are examined. For example, author studies the influence of any other components of ground motion on the one which should be measured by the apparatus. Some remarks for measurement of a suddenly beginning motion are also added. Reviewer thinks, however, the theoretical considerations just mentioned are well known by seismologists and manufacturers, at least in Japan. In the latter half of the paper, a detailed description of crystal accelerometers and velocity meters of electromagnetic type is given. An electric network with filterlike circuit is devised. The damping intensity can be adjusted in a wide range, though the vibrometer itself is practically undamped. After being amplified as much as 10^5 times with an RC-amplifier, the voltage generated by the vibrometer is applied to a cathode-ray oscillograph or a set of crystal galvanometers with sensitivity of 1 mm/volt per 2 m scale. The apparatus is used for measuring vibration of structure and also for seismic prospecting.

Tsuneji Rikitake, Japan

4086. Bondar', N. G., On the approximation of fundamental functions and of the function of small dynamic motions of rod systems (in Russian), *Prikl. Mat. Mekh.* **15**, 2, 207–226, Mar.-Apr. 1951.

General solution of the problem of a rod-system's forced vibrations contains fundamental functions and eigenvalues of the system's kernel. Of course, one knows them exactly in simple cases only and engineers, as a rule, must calculate with approximations. In the first part of his paper, author shows how to estimate fundamental functions and eigenvalues of any order. In the second part, a particular solution of general equation of forced vibrations is found by means of successive approximations.

All considerations of the first part base on the following theorem: "Let λ_ν be characteristic values, $\varphi_\nu(x)$ fundamental functions of the rod-system's influence function with multiplicities q_ν, respectively; let $K_\nu(x,s)$ be successive iterations of the kernel $K(x,s) = \sum_k^\infty \varphi_i(x)\, \varphi_i(s)/\lambda_i$. Then it is $\lim \lambda^m{}_k K_m(x,x) = \sum_k^{k+q_k-1} \varphi_i{}^2(x)$ absolutely and uniformly for each natural k and for $m \to \infty$." For $q_k = 1$, one obtains interesting estimates of technical value from here; of course, they suppose knowledge of all eigenvalues and fundamental functions of lower order than the k-th. Author then deduces some formulas for approximating characteristic functions of any order and utilizes the results for estimating higher functions $\varphi_k(x)$ without use of lower functions. Then he gives a survey of his own experiences on the degree of his approximations and applies former results to a technical example.

The second part of the paper begins with a fine application of successive approximations method to solving general equation of forced vibrations; also formulas are given for estimating the function of small dynamic motions of rod systems. The author deduces further conditions under which one can express particular integral of the equation, mentioned above, in a closed form. The theoretical results are applied to calculation of forced vibrations without damping of a girder.

Reviewer considers the article as an enrichment of the theory of rod systems and of their practical computation. The second part forms a fine whole from the mathematical point of view; the first contains some slight vaguenesses. The summation in the formula (2.1) should probably go up to $k + q_k - 1$ only and not to $k + q_k$. Relations (5.1) can easily be deduced for odd values of k; the case of the even k is more difficult and perhaps special attention should be paid to it.

The degree of the approximations, given in the paper, is in general satisfactory for technical purposes, and reviewer recommends the article to engineers and physicists.

V. Vodička, Czechoslovakia

Wave Motion, Impact

(See also Revs. 4084, 4098)

4087. Weitz, M. and Keller, J. B., Reflection of water waves from floating ice in water of finite depth, *Comm. pure appl. Math.* **3**, 3, 305–318, Sept. 1950.

Paper treats surface waves in water of arbitrary, finite depth which are incident at an arbitrary angle on the line of separation between two half planes in the surface. The surface conditions may correspond to floating matter (ice) of different constant surface densities on the two half planes, to floating matter on one half plane and a free surface on the other, or to a dock on one half plane and a free surface on the other. By means of a Green's function, an integral equation is obtained for the potential which is similar to but not exactly of Wiener-Hopf type. It is solved by a modification of the Wiener-Hopf technique to yield an infinite series. Questions of boundedness, uniqueness, and number of solutions are discussed under three cases corresponding to inequalities on the parameters characterizing the surface conditions.

J. J. Gilvarry, USA

4088. Malvern, L. E., The propagation of longitudinal waves of plastic deformation in a bar of material exhibiting a strain-rate effect, *J. appl. Mech.* **18**, 2, 203–208, June 1951.

Paper may be regarded as supplementing author's previous paper [AMR **4**, Rev. 2372] which dealt with the same topic in general terms, largely from a mathematical standpoint. In this paper the theory is applied to the particular case of a material for which the law of flow is given by: (*plastic strain rate*) proportional to (*instantaneous stress*) minus (*stress corresponding to the same strain in a static test*). Using numerical values which agree approximately with experimental values for hardened Al, the equations of motion are integrated and the results shown in a family of curves giving (*stress-strain*) relationships at rates of strain varying from 0 to 400 (sec)$^{-1}$. These results are used to study the propagation of plastic waves initiated in semi-infinite bars by impact first, when the velocity of the struck end rises instantaneously from zero to a finite value and, secondly, when this rise is exponential. The results are summarized in a series of graphs in which the difference between a zero and a finite strain rate is clearly indicated. Comparison of calculated and experimental results leads to the conclusions stated in author's previous paper.

R. M. Davies, Wales

4089. Kusukawa, K., On the theory of shock waves produced by a rigid wedge moving through an elastic medium with supersonic velocities, *J. phys. Soc. Japan* **6**, 3, 163–165, May-June 1951.

Disturbances in an elastic body can be propagated by either of two types of waves: the "irrotational" and the "equivoluminal," each with different characteristic velocities. Author shows first that, when the strains are very small and the velocity of the wedge is supersonic, there appear in each of these two cases progressive wave fronts across which discontinuities in stress, strain, and density may occur, analogous to the appearance of shock waves in compressible fluid flow. Secondly, the principles of conservation of mass, momentum, and energy applied to the transition across these wave fronts give the "jump conditions" for the values of stress, strain, and density.

The jump in value of the strain normal to a shock front is worked out numerically for several Mach numbers and seems to be a minimum for $M = 1.36$.

W. Freiberger, Australia

4090. Kusukawa, K., On the theory of shock waves produced by a rigid cone moving through an elastic medium with supersonic velocities, *J. phys. Soc. Japan* **6**, 3, 166–167, May-June 1951.

The method of the preceding paper is applied here to the corresponding three-dimensional problem.

W. Freiberger, Australia

Elasticity Theory

(See also Revs. 4112, 4131)

4091. Massonnet, Ch., Graphical resolution of general problems of plane elasticity (in French), *Bull. Centre Étud. Constr. génie civ. Hyd. Fluviale* **4**, 3–183, 1949.

The problems considered are those occurring in the theory of elastic plates, loaded in their own plane. The simplest one is that of determining the state of stress in a simply connected plate, bounded by the contour C and loaded along C by the load

system $t(s)$, s being measured along C. The principle of the proposed method of solution is that of introducing a fictitious load system $p(s)$, defined as follows. To each elementary fictitious load $p(s)\,ds$ is attributed a stress system, i.e., the one that would exist in a semi-infinite plate, bounded by the tangent to C in s and on the same side as the actual plate under the action of the load $p(s)ds$. If necessary, the stress system, the state of simply radial stress, has to be extended outside the semi-infinite plate. This stress system induces known stresses along C. Superposition of these elementary systems must produce stresses along C in accordance with $t(s)$. This leads to a vector-integral equation of the Fredholm type and second kind. This idea is not new, but it is extended in many directions.

The most obvious method of solving the integral equation is by iteration, i.e., by inserting an approximation to $p(s)$ under the integral sign and finding a better approximation in this way. If $t(s)$ is a hydrostatic (uniform normal) pressure, $p(s) = {}^1/_2 t(s)$, but then this iteration does not converge. Moreover, it is necessary that $t(s)$ be in equilibrium, though $p(s)$ does not need to be. In the case that C is a circle, it is shown that there is only one solution $p(s)$ in equilibrium to a given $t(s)$, but that also there is one solution $p(s)$ inducing no stresses but giving rise to a prescribed resulting force of couple. These difficulties are overcome by using a more refined iterative scheme that converges always, and always to the solution in equilibrium. To avoid the necessary laborious calculations, an apparatus is described in detail that performs semi-automatically the intricate vector summations. As example, stresses in a joint of a Vierendeel truss under tension and bending are calculated and shown.

In the case of multiply connected plates, the elementary stress system to be used is another one, i.e., the one induced by a concentrated force in a completely infinite plate. The corresponding problems of prescribed displacements and mixed boundary conditions are noted. Appendixes include the analogy with problems in fluid mechanics, and many rigorous mathematical investigations completing and extending the foregoing chapters. A. van Wijngaarden, Holland

4092. Ugodchikov, A. G., Determination of stresses arising from forced-fit of several round disks in a plate bounded by a special curve (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **77,** 2, 213–216, Mar. 1951.

With reference to several previous Russian papers, author expresses generally the conditions on the boundary of a plate and on the boundaries of round disks pressed in the plate. Functions for calculation of stresses are given in general form. Results of a solution of one and two disks pressed in a plate with curved boundary free from stresses are indicated.

Z. Bažant, Czechoslovakia

4093. Föppl, O., I. A new elasticity theory, based on the natural moduli E_0 and G. II. Bending vibrations of running rotors. III. Discrepancies between theory and practice (in German), Braunschweig. Friedr. Vieweg & Sohn, *Mitt. Wöhler-Inst.*, no. 44, 48 pp., 1950. DM 3.60.

4094. Filonenko-Borodich, M. M., Problem on the equilibrium of an elastic parallelopiped with given loads on its sides (in Russian), *Prikl. Mat. Mekh.* **15,** 2, 137–148, Mar.-Apr. 1951.

Problem of Lamé is resolved by Castigliano's theorem of least work. Six components of stresses in a point of the body are expressed by basic values satisfying equations of equilibrium and boundary conditions. To them are added corrective values containing a sufficient number of linear parameters to permit an arbitrary approximation of the exact values of stresses. The corrective values are expressed by Maxwell's stress functions in the form of series containing cosinus binomials. Basic stresses are determined in concordance with loading. For a parallelopiped compressed on two opposite sides by equal and opposite forces distributed according to chosen law, the solution is shown generally and also numerically for a load distribution governed by function of cosinus. In first approximation, calculation uses three parameters whose values are obtained from conditions that make the derivative of strain energy equal to zero. The second approximation, using twelve parameters, shows sufficiently equalized stresses in the median section parallel to compressed sides to account for much variable external compression. General solution is given for loads on two opposite sides in equilibrium, but not equal. Z. Bažant, Czechoslovakia

4095. Edwards, R. H., Stress concentrations around spheroidal inclusions and cavities, *J. appl. Mech.* **18,** 1, 19–30, Mar. 1951.

Exact closed solutions are obtained for (a) the distribution of stress around a spheroidal inclusion or cavity in an elastic body which is in an arbitrary uniform state of stress at infinity; (b) the thermal stress distribution arising from distinct uniform temperature charges applied to the spheroidal inclusion and the surrounding medium.

The equations are solved in terms of three harmonic functions in the formulation due to Sadowsky and Sternberg [AMR **1,** Rev. 788] based on Boussinesq's method. Five basic sets of solutions, corresponding to different loading conditions at infinity, are found, and the stress distributions determined by linear superposition.

For the case of uniaxial tension at infinity the value of the stress component in the direction of tension is evaluated at a pole and at two equatorial points of the spheroid, for different values of shape ratio and material parameter. The normal stress at these three points is determined also for the case when the body and inclusion are at different uniform temperatures, have different elastic properties, and there is no stress at infinity.

W. Freiberger, Australia

4096. Sen, B., Stresses due to nuclei of thermoelastic strain in a thin circular plate, *Bull. Calcutta math. Soc.* **42,** 4, 253–255, Dec. 1950.

Paper gives, in simple closed form, elastic thermal plane stress due to hot element anywhere in an otherwise cold disk, using author's special method for two-dimensional boundary-force problems, and known result for corresponding thermal stress problem of infinite plate. J. N. Goodier, USA

4097. Föppl, H., The evaluation of macroscopic residual stresses in cylindrical bars, *J. Iron Steel Inst. Lond.* **168,** part 1, 15–22, May 1951.

Paper deals particularly with residual stresses produced in plastic deformation of surfaces by shot-peening or surface-rolling. Author measures residual stresses in shot-peened bars destroying, by etching away, the plastical deformed part (cylindric surface), until only the elastic part (core) is retained. Length changes vs. removed thickness rises first linearly, passes a maximum point, and gradually approaches the horizontal. Author's new experiments are more accurate than previous ones [*Mitt. Wöhler-Inst.*, 112, 557, 1949, Braunschweig.]. Considering some recent experiments in the straining of metals, an "intermediate layer" is introduced, a discontinuity between the biaxial plastically stressed surface and the triaxial elastically stressed core in which a "stress reversal" in the plastic deformation takes place. Residual stresses are calculated from strains in principal

axes and Hooke's law, introducing strain ratios for surface and core, $\delta_s = e_{ts}/e_{as}$ and $\delta_k = e_{rk}/e_{ak}$. Calculation is possible even if the strains are measured in only one direction. Author's equation reduces to Heyn-Sachs if Poisson's ratio is 0 (both strain ratios are zero, too). Calculated core stresses are much greater in axial than in radial direction, implying negative radial or tangential strain, i.e., $\delta_k < 0$. In the surface $\delta_s > 0$. Author assumes both δ constant (error 10%. better than with $\nu = 0$). The assumed δ are altered so that both conditions of equilibrium are fulfilled. Sachs' formula only fulfills one, giving inaccurate values for tangential and radial stresses. As it does not consider the stresses in the third axis, Sachs' formula is only valid for ring-shaped cross section in which the radial stress can be neglected. Author says both reasons justify the removal of cylindrical layers instead of boring them out. Future research will show if it is preferable to replace Sachs' formula by a more complicated calculation or to measure directly the stresses in the core by other experimental techniques. The calculated values show the plausibility of assuming a biaxial state in the surface and a stress reversal at 0.013 in. depth. A very fine and interesting paper.

Leonardo Villena, Spain

4098. Satô, Y., Boundary conditions in the problem of generation of elastic waves, *Bull. Earthq. Res. Inst. Tokyo Univ.* **27**, parts 1–4, 1–9, Jan.-Dec. 1949.

Author considers the problem of the generation of elastic waves in which the boundary conditions on the surface of a sphere are given in general form. Explicit solutions of the elastic equations are obtained by writing the equations in spherical polar coordinates and determining the expansions in spherical harmonics of the components of stress. The solution so obtained is not discussed in any way.

Ian N. Sneddon, England

Experimental Stress Analysis

(See also Rev. 4171)

4099. Bennett, J. A., A study of fatigue in metals by means of X-ray strain measurement, *J. Res. nat. Bur. Stands.* **46**, 6, 457–461, June 1951.

Determination of lattice strain was made under static bending moments accurately measured and after various amounts of fatigue stressing. Specimens were heat-treated in vacuum. Changes in the diffraction angle at two different incidence angles of x-ray beam were obtained as described previously [Bennett, J. A., *Rev. sci. Instrum.* **20**, p. 908, 1949]. Radius of $K\alpha_1$ diffraction ring vs. incidence angle, lies approximately on straight line and gives diffraction angle 2θ. Author takes as lattice strain the difference in 2θ at 0 and 45° incidences, plots it vs. bending moments and selects the results in some way. This average slope usually decreases with number of stress cycles, independently of fatigue cycles; plastic deformation appears less under static stress than under the prior fatigue stress.

Leonardo Villena, Spain

4100. Ekstein, H., and Siegel, S., Achromatization of Debye-Scherrer lines, *Nat. adv. Comm. Aero. tech. Note* 2355, 23 pp., Apr. 1951.

A method is described for reducing the width of a Debye-Scherrer line produced by diffraction from a polycrystalline medium, if this width is due to the spectral impurity of the primary characteristic radiation. In this method, a diverging polychromatic beam is allowed to fall on the plane surface of a single crystal. The beam diffracted by this crystal will diverge and will contain a bundle of rays whose wave-length range corresponds to the finite spectral width of the characteristic radiation. The polycrystalline sample is mounted normal to this bundle. It is shown that the different wave lengths diffracted by the sample can be brought to a narrow focal spot.

From authors' summary by C. O. Dohrenwend, USA

4101. Bühler, H., and Schreiber, W., Compensation of measurement scatter in the boring-out method of George Sachs for determination of residual stresses in rods and tubes (in German), *Metall* **5**, 3/4, 53–57, Feb. 1951.

Some authors suggest that the test data obtained in the boring-out method be approximated by power functions. In many instances the longitudinal strain λ and the tangential strain ϑ follow satisfactorily simple parabolas, as follows: $\lambda = a_1 f + a_2 f^2$, $\vartheta = b_1 f + b_2 f^2$ where f is the variable cross-sectional area of the bore. The constants in these equations must be determined by the method of least squares. The longitudinal stress σ_L, the tangential stress σ_T, and the radial stress σ_R, are then given by the following equations:

$$\sigma_L = (E/1 - \mu^2)(A_0 + 2A_1 f + 3A_2 f^2)$$
$$\sigma_T = (E/1 - \mu^2)(B_0 + 3B_1 f + 5B_2 f^2)$$
$$\sigma_R = (E/1 - \mu^2)(B_0 + B_1 f + B_2 f^2)$$

where

$$A_0 = f_b(a_1 + \mu b_1), \qquad 2B_0 = f_b(b_1 + \mu a_1)$$
$$A_1 = f_b(a_2 + \mu b_2) - (a_1 + \mu b_1), \; 2B_1 = fb(b_2 + \mu a_2) + (b_1 + \mu a_1)$$
$$A_2 = -(a_2 + \mu b_2), \qquad 2B_2 = -(b_2 + \mu a_2),$$

and f_b is solid cross-sectional area of cylinder, E elastic modulus, μ Poisson's constant. The above equations yield values which conform to the equilibrium conditions for the longitudinal and tangential stresses, as well as to the conditions $\sigma_R = 0$ at the surfaces and $(\sigma_R = \sigma_T)_{R=0}$ for the solid cylinder. An example for the procedure has been presented. If parabolas of the second order do not present the experimental data sufficiently closely, a third power function may be used.

George Sachs, USA

4102. Frisch, J., and Thomsen, E. G., Residual grinding stresses in mild steel, *Trans. Amer. Soc. mech. Engrs.* **73**, 3, 337–342, Apr. 1951.

Formula giving residual stresses in the work-hardened layer in a bar is derived, based on Davidenkow and Shevadin's theory. According to E. K. Henriksen, this formula is incomplete, as it does not give the values of stresses over the cross section of the bar outside of the work-hardened layer.

SAE 1020 hot-rolled, fully annealed bars were ground to a depth of 0.0003 to 0.003 in. in a surface grinder, cleaned, and etched with 5 to 15% nitric acid in water to remove stressed layers of the order of a few 0.0001 in. The change in the deflection and the remaining thickness of the bar were measured after each etch and the residual stresses calculated.

Results show: Maximum surface stresses are well above the original yield point of the material; thickness of layer containing residual stresses increases with increasing depth of grind; maximum surface stresses are smaller for heavier cuts than for light; depth of surface layer deformed by grinding increases with depth of cut.

Dimitri Kececioglu, USA

4103. Landwehr, R., Light interference as a means for treating strength of materials problems (in German), *Z. Ver. dtsch. Ing.* **93**, 16, 452–454, June 1951.

Paper deals with application of light interference to such problems as bending of thin, flat plates, determining elastic constants of model materials such as glass, and finding the isopachics in stressed plates by measuring thickness variations. Author de-

scribes experiments on glass models of centrally loaded thin circular plates freely supported on the outer edge or three equally spaced points, and thin square plates freely supported at the corners. The deflection of such plates is determined from a light-interference fringe pattern. The plate surface is polished flat and placed in a loading apparatus parallel to a reference optical flat. The fringes produced by interference of light reflected from the deflected plate and the reference flat represent lines of equal deflection. Author reports good agreement of experimental results with exact analytical solutions.

Using the same circular glass disk and the same load but two different methods of support, the optically determined maximum deflections together with the analytical solutions can be used to calculate the elastic constants of the glass. Accuracy of ±1.1% can be expected. Nicholas Sag, Australia

Rods, Beams, Shafts, Springs, Cables, etc.

4104. **Pode, L., Tables for computing the equilibrium configuration of a flexible cable in a uniform stream,** *David W. Taylor Mod. Basin Rep.* 687, 31 pp., 4 tables, Mar. 1951.

Expressions for the coordinates of the equilibrium configuration of a flexible cable immersed in a uniform steady stream are obtained analytically and, from them, tables of functions useful in plotting such configurations are computed and given to four decimal places. It is assumed that the configuration lies entirely in one plane, that the cable is essentially round, and that neither the cable weight nor its tangential drag in the stream is negligible.

The tabulated functions, as is illustrated by examples, facilitate determination of the towing tension and of the shape of the curve between points conforming to certain "end conditions" for cables towed in air or water. None of the examples provides for a transition curve between the points which satisfy the prescribed end conditions and the points at which the end forces are actually applied as, for instance, when an aircraft cable carrying a streamlined weight is paid out through a vertical tube which protrudes a short distance into the airstream.

The range of the tables is adequate for most practical problems involving cables of nautical or aeronautical sizes.

Joseph S. Newell, USA

4105. **Håkansson, Å., Calculation of cylindrical and conical shells under external overpressures** (in Swedish), *Tekn. Tidskr.* 81, 13, 261–263, Mar. 1951.

As a basis for his work, author uses the equation for critical load for externally loaded cylindrical shells developed by von Mises. For certain dimension ratios and for definite values of modulus of elasticity and Poisson's ratio, he develops expression for minimum value of critical load. Nomographic charts are given (factor of safety 5) to expedite solution. Charts are given to correct for change of factor of safety, change of modulus of elasticity, and temperature change. Both stiffened and unstiffened shells are treated. Numerical example demonstrates easy use of material presented. Robert B. B. Moorman, USA

Plates, Disks, Shells, Membranes

(See also Revs. 4091, 4096, 4103, 4105, 4121, 4124, 4141, 4155)

4106. **Nash, W. A., Bending of annular elliptical plates loaded by edge moments,** *Bull. Calcutta math. Soc.* 42, 4, 189–198, Dec. 1950.

Article considers the linear bending of a thin plate bounded by two confocal ellipses and loaded by normal bending moments around both boundaries. A series solution is indicated using elliptic functions. Three problems are discussed: (1) Known deflections and bending moments on both boundaries; (2) known bending moments on both boundaries and a known deflection on the outer boundary with the inner boundary free to deflect, i.e., no vertical reaction at the inner boundary; (3) like (2) except that that outer boundary is free to deflect and the inner boundary has the specified deflection. This article is an extension of an earlier paper [AMR 4, Rev. 2416] by the same author.

William B. Stiles, USA

4107. **Münz, H., A method of integration for the calculation of bending stresses of axisymmetrical shells under axisymmetrical load, I** (in German), *Ing.-Arch.* 19, 2, 103–117, 1951.

Purpose of this thesis is to find equations which are easier to integrate numerically than those derived by H. Reissner and E. Meissner. Transition to canonical variation theory leads to variables in the equations with a real significance, this as distinct from the variables in the equations of Reissner and Meissner. With aid of a known integral, integration problem can be reduced from sixth to fourth order, which explains why Reissner and Meissner could derive simultaneous differential equations of second order.

First part deals with derivation of canonical equations only. Second part will show that the canonical form is advantageous for numerical integration. M. Botman, Holland

4108. **Hill, R., A theory of the plastic bulging of a metal diaphragm by lateral pressure,** *Phil. Mag.* (7), 41, 322, 1133–1142, Nov. 1950.

In the reviewer's opinion, this paper offers the first satisfactory theoretical treatment of the problem indicated in the title. The mathematical technique used by author is essentially a perturbation procedure with the ratio of the thickness to the radius of the plate as parameter. The resulting formulas apply to a perfectly plastic material as well as to materials with linear strain hardening and seem to agree well with experimental data.

W. Prager, USA

4109. **Holms, A. G., Jenkins, J. E., and Repko, A. J., Influence of tensile strength and ductility on strengths of rotating disks in presence of material and fabrication defects of several types,** *Nat. adv. Comm. Aero. tech. Note* 2397, 39 pp., June 1951.

This report, one in the voluminous series based on the work of the NACA Lewis Flight Propulsion Laboratory, covers a group of tests on parallel-sided disks under centrifugal stress.

Two styles of disks were tested, one with and the other without a bore hole, by spinning to destruction, and the maximum stress at bursting was calculated. Three materials were tested, cast beryllium copper, forged age-hardenable 18-8 stainless steel, and a cast magnesium-base alloy. These materials were heat treated to obtain varying ultimate tensile strengths and ductilities. The corresponding types of defects were shrink porosity, laminar type irregularities, and eutectic melting, but the tensile and ductility measurements were made in sound metal specimens. These tests indicated a definite relation between tensile strength and ductility in a disk, in the presence of material and fabrication defects in the steel and copper disks, but no significant variation in the magnesium-base alloy. Irregularities resulted in a loss in disk strength from 23 to 58% compared to previously observed disks of sound metal.

These tests represent a valuable contribution to the information on the strength of disks as used in turbines and compressors. The tests, however, were somewhat limited in number

and scope, and so it is probably well not to draw too general conclusions from them. A. O. White, USA

4110. Bijlaard, P. P., Investigation of the optimum distribution of material in sandwich plates loaded in their plane, including plates with varying modulus of elasticity, *Cornell Aero. Lab. Rep.* SA-247-S-8, 53 pp., Mar. 1951.

Author derives expressions for optimum selection of materials for sandwich-plate construction. Assuming that face and core materials are isotropic and have the property that density is proportional to elastic modulus, author computes ratio of core modulus to face modulus, which, for a given weight per unit area of sandwich plate, gives greatest buckling load. This calculation is carried out for fixed face-to-core thickness ratio, as well as for fixed total thickness. Various edge and loading conditions are assumed. A five-layered laminate with three constant moduli is also treated.

Critical load formula on which optimum calculations are based is from another paper by same author. It is an approximate formula, valid when buckling is not of wrinkling type. Buckling wave length must be long compared to thickness of sandwich.

H. J. Plass, USA

4111. Hemp, W. S., Influence of the testing machine on the flexural failure of panels, *Aero. Res. Counc. Lond. Rep. Mem.* 2539, 6 pp., Oct. 1945, published 1951.

Pin-ended length of compressed panels is function of (a) flexibility of testing machine structure, and (b) flexibility of plattens. Author finds (a) experimentally by compressing a pin-ended strut, and calculates (b) from theory for compressed elastic half space. Numerical example is worked out and results are presented graphically. H. D. Conway, USA

4112. Tarabasov, N. D., State of stress of an elliptic plate with several pressed-in round disks (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **77**, 1, 33–36, Mar. 1951.

Conditions at ellipse and initial displacements of disks are assumed to be given arbitrarily. Author applies method developed by D. I. Sherman [title source, **31**, 4, 1941], reducing the plane problem to finding two functions of complex variable, each being regular in its respective region.

General solution, obtained in form of series, is illustrated by numerical example of two equal and symmetrically situated disks. The corresponding stress distribution along the axes is represented graphically. J. M. Klitchieff, Yugoslavia

4113. Paschoud, J., Calculation of cylindrical tubes of revolution with variable thickness, stressed in circular flexure (in French), *Bull. tech. Suisse Rom.* **77**, 2, 13–20, Jan. 1951.

Paper deals with problems in which the walls of the tube are bent with axial symmetry. Formulas and graphs of various influence coefficients are given for cases of linear and parabolic variation of wall thickness. Method of use is illustrated by numerical examples. W. S. Hemp, England

4114. Kavanaugh, E. S., and Drinkwater, W. D., Torsional strength of stiffened D-tubes, *Nat. adv. Comm. Aero. tech. Note* 2362, 47 pp., May 1951.

Note covers a series of torsional tests on stiffened D-tubes of Alclad 24S-T3 aluminum alloy having a cross section similar to the NACA 0012 airfoil section and a closing web at 30% of the chord. In more general terms, the cross section consisted of half of an approximate ellipse with axis of 7.2 in. and 36 in., cut through minor axis. Stiffeners consisted of ribs and stringers. Skin thickness, web spacing, and stringer spacing were varied, and results were evaluated with respect to these variables. In addition, rib thickness, web thickness, stringer size, and rivet size increased as the skin thickness increased. The parameters used in evaluating the results are similar to those used by L. H. Donnell (in his analysis of the problem of buckling of a shell subject to torsion) except that an additional parameter, the chordwise length of the largest panel, was introduced. Authors found better argument using skin thickness with an exponent of 1.75 instead of 2, as used by Donnell. Somewhat surprisingly, buckling failure due to torsion of unstiffened D-section and cylindrical shells appears to be predictable by the same equation. In addition to failure, using ultimate torsional load as a criterion, load at first visual evidence of buckling was obtained. Also, measurements of unit twist and unit strain were obtained. Article should be of interest to anyone working with complex shapes in torsional loading. Everett C. Rodabaugh, USA

4115. Boley, B. A., Kempner, J., and Mayers, J., A numerical approach to the instability problem of monocoque cylinders, *Nat. adv. Comm. Aero. tech. Note* 2354, 45 pp., Apr. 1951.

Buckling loads of three cylinders with widely different characteristics, subjected to pure bending, have been calculated by means of two numerical methods. Authors consider only the most highly compressed portion of the cylinder. The first method makes use of a 14-row determinant, the second of a 10-row one. Experimental results show a discrepancy of 20–30% between theoretical and experimental values. A procedure similar to the first method was applied to a cylinder with a cutout. Here there is a greater deviation between theory and experiment.

M. Kuipers, Holland

4116. Monge, F. G., Method of Professor Johansen for calculation of plates (in Spanish), *Informes Construc.* **3**, 27, 8 pp., Jan. 1950.

Article treats briefly Johansen's theory of lines of rupture. Four numerical examples of plates (3 rectangular, 1 circular) with uniform load and different boundary conditions illustrate the main ideas. Comparisons with elastic theory point out the reducing of moments by Johansen's method and its economical advantages, also simplicity of calculation.

Stating of fundamentals is too condensed to be clear. Important contributions by Johansen are not mentioned [*Int. Assn. Bridge Struct. Engng.* **1**, 1932, and Third Congress, 1948, Final Report; also Doctor Thesis (in Danish), *Techn. Univ. Denmark*, 1943]. Arturo M. Guzmán, Argentina

4117. Ballet, M., and Mallet, G., Stresses in tube-plates of boiler collectors (in French), *Bull. Assn. tech. marit. aéro.* no. 49, 477–489, 1950.

Paper gives six graphical tables (obtained through photoelastic experiments) for stress calculation in a pierced rectangular plate submitted to uniform pressure on two opposite sides in its plane. Experiments were for the purpose of finding the stresses in the back plate of a fire-tube boiler and are very accurate.

Reviewer notes that, except for some research by M. Legendre, no reference is made to previous papers on the same or similar subject, and, consequently, no comparison is made with results of other authors. Actually, the reported experiments are in sufficient agreement with previous researches [see, for theoretical developments, Kirsch, *Z. Ver. dtsch. Ing.*, 1898; Pöschl, Th., *Z. angew. Math. Mech.* **1**, 1921; Weber, C., ibid., 1922 and 1941; Hutter, A., ibid., 1942, etc; and for experimental stress analysis, Hannig, A., *Forschung.*, 1933; Siebel and Kopf, *Forschungsheft* n. 369, 1934; Coker and Filon, "Photo-elasticity," 1931, chap VI, etc.] Giulio Supino, Italy

4118. Mezhlumyan, R. A., Flexure and torsion of thin-walled cylindrical shells beyond the elastic limit (in Russian), *Prikl. Mat. Mekh.* **14, 3,** 253–264, May–June 1950.

A thin-walled cylindrical shell is assumed to be compressible and to obey in the plastic range a stress-strain law of deformation with hardening. Paper is confined to a reduction of the equilibrium equations to a system of fourth-order ordinary differential equations in terms of displacements. For the original formulation of the problem the reader is referred to the work of Vlasov ["Thin-walled elastic bars," Moscow-Leningrad, Gosstroĭizdat, 1940] and for methods of solving the plastic-elastic boundary-value problem to the work of Ilyushin [AMR **4**, Rev. 2913].

Courtesy of Mathematical Reviews H. I. Ansoff, USA

4119. Craemer, H., Calculation of compound systems by iteration (in German), *Öst. Ing.-Arch.* **4,** 5, 350–354, Nov. 1950.

The "compound system" referred to is a thin-walled prismatic shell in flexure; for numerical example author uses an octagonal subjected to wind load. Bending stresses are found for each face acting independently; at edges, adjoining faces, in general, give different stresses. These differences are resolved by a method analogous to the Cross method for continuous beams, etc. Example bears out previous work showing that the flexure formula is invalid for this type of structure as a whole. References consist of only the author's own publications.

A. D. Topping, USA

Buckling Problems

(See also Revs. 4114, 4115)

4120. Jung, H., A contribution to the computation of buckling loads (in German), *Z. angew. Math. Mech.* **31,** 4/5, 142–148, Apr.–May 1951.

The Fourier transform method is employed to obtain the critical buckling loads of a rod of variable cross section, elastically restrained at its ends against rotation and deflection. Method is exact for simple cases (prismatic bars) and is extended to more complicated ones by a convergent iteration procedure due to Collatz. L. E. Goodman, USA

4121. Lindholm, E., Buckling of a circular saw with symmetrical temperature distribution (in Swedish), *Tekn. Tidskr.* **80,** 11, 243–247, Mar. 1950.

Author states that because of the high temperature at the periphery, caused by frictional forces, the blade of a circular saw may become unstable. Precautions may be to heat the central part of the blade or to give this part initial stresses. The differential equation for the deflection (Laplace) is solved, and buckling temperatures are found for the other and inner part of the saw blade. Some experimental data are given.

C. V. Bernhardt, Norway

4122. Teichmann, F. K., Wang, C.-T., and Gerard, G., Buckling of sandwich cylinders under axial compression, *J. aero. Sci.* **18,** 6, 398–406, June 1951.

Using accepted equilibrium equations of homogeneous cylinder and Reissner's stress-displacement relations for curved sandwiches, differential equations are established for compressive buckling of sandwich cylinders. Core is assumed infinitely stiff in radial direction, and faces are treated as membranes.

It is found that if core has a shear stiffness G_c greater than $10\,E_f t/a$ (where f is face modulus, t face thickness, and a radius) buckling load is not affected by shear flexibility. When shear stiffness is less than one tenth of this value, buckling load per inch circumference is $(h + t)G_c$ (where h is core thickness).

Tests are reported for eleven cylinders having cellular cellulose acetate cores in the low stiffness range. Results are in reasonable agreement with theory. S. B. Batdorf, USA

4123. Cicala, P., On plastic buckling of plates and a theory of plastic slip, *J. aero. Sci.* **18,** 4, 285–286, Apr. 1951.

Finite or deformation theories of plasticity are in good agreement with experimental data on plastic buckling of plates but have been found unsatisfactory for abruptly changing loading paths. Incremental or flow theories are better for abruptly changing loading paths but are difficult to reconcile with buckling experiments. Slip theory of plasticity introduces possibility of agreement with both types of experiment because in it strain increments are not linear functions of stress increments.

Present note uses slip theory to calculate compressive buckling and approximate postbuckling behavior of long-hinged flanges. Twisting stiffness of flange in plastic compression is calculated and shown to decrease with increasing ratio of increment of load to increment of twist. This permits buckling and postbuckling analysis of flange to be carried out in manner parallel to that for Shanley buckling of columns. Buckling load—i.e., load at which flange begins to bend—is found to coincide with that given by deformation theory (and experiment), and a reduced modulus is found for maximum load.

That slip theory agrees with deformation theory for buckling of flanges has been noted previously by reviewer [AMR **3,** Rev. 1924]. However, author's work on postbuckling behavior of hinged flanges appears to be the first numerical application of slip theory to stress analysis. Although formula found for maximum load is in poor agreement with experiment, due largely to use of small deflection theory in the structural analysis [see, for example, AMR **3,** Rev. 1888], reviewer considers development of analytical methods for handling such problems by slip theory to be a significant contribution. S. B. Batdorf, USA

4124. Kuenzi, E. W., and Ericksen, W. S., Shear stability of flat panels of sandwich construction, *For. Prod. Lab. Rep.* 1560, 42 pp., Mar. 1951.

Behavior of sandwich panels under pure shear load is studied both theoretically and experimentally. Edge conditions studied include both simply supported edges and clamped edges; finite rectangular plates and infinitely long panels. A core stiffness parameter S is defined in terms of the facing thickness, the face modulus, the core thickness, and the core shear modulus. The critical stress may then be given in terms of a stress factor L_0 for a plate with infinite shear stiffness in the core, the core stiffness parameter S, and a quantity S_1 which depends on the aspect ratio of the panel and the edge conditions when S is less than S_1. When S is greater than S_1, the critical stress factor L is simply given by $1/S$. Method for determining critical shear stress beyond the proportional limit is also suggested. Results of tests on 28 square panels of symmetrical sandwich construction are reported and these are in good agreement with the prediction of the theoretical analysis. Conrad C. Wan, USA

Joints and Joining Methods

4125. Goelzer, A., Internal strength of fillet welds (in French), *Publ. int. Assn. Bridge Struct. Engng.* **10,** 37–68, Nov. 1950.

Analysis of normal and parallel fillet welds is based on limit of elasticity defined by "intrinsic curve" which expresses normal and

shear stress on plane of first yielding in terms of elastic limits in tension and compression.

Welds are assumed to have a right triangle cross section. For normal fillet weld, plane stress is assumed and stress distributions satisfying conditions of elasticity are given for two locations of the resultant load. In parallel fillet weld, the stress distribution is pure shear. On the basis of these distributions, the allowable load in the weld is calculated, allowing for a factor of safety with respect to first yielding.

Reviewer believes results are of limited value due to idealized weld shape and the assumption of elastic limiting conditions.

W. O. Richmond, Canada

4126. Jaeger, H. E., Shrinkage and shrinkage phenomena resulting from arc welding (in French), *Rev. Soud.* **6**, 2, 83–95, 1950.

Author mentions experiments made by Léon Bibber [*Welding J.*, 1948], Stocks and Thurston [*Welding*, 1948–49], Field, Kennedy and Gerritsen (no literary references). He shows that a number of shrinkage phenomena (stresses and deformations) can be explained by the fact that the contraction of a bar of steel cooled down from 1000 C to 20 C is greater than the expansion by the foregoing heating process. A boss in a steel plate can, therefore, be removed by local heating of the plate. In welding of great constructions (e.g., ships), the consecutive order of the separate weldings must be chosen with regard to the greatness of the residual stresses and deformations. Also, the speed with which the welding is done and the cooling rate are important.

A. R. Holm, Denmark

4127. Soete, W., and VanCrombrugge, R., Study of the fatigue resistance of welded joints (in French), *Rev. Soud.* **6**, 4, 199–219, 1950.

Paper is part of a series on the fatigue of welded connections. Double fillets of various throat dimensions (from $^{3}/_{16}$ to $^{11}/_{16}$ in.) were investigated under repeated tensile load in the as-welded and in annealed state. The results are compared with those of other investigators, especially with those of Bierrett.

Authors characterize the different double fillets with the dimensional proportions $2\ d/e$ and $2\ f/e$, d the leg of the fillet weld, f the root penetration, and e the distance between the roots at the base of the joint. A definite change in the type of failure and in strength under cyclic loading can be observed when a certain value of $2\ d/e$ is present.

In the case of double fillet welds with no root penetration, i.e., $2\ f/e = 0$, this change occurs at $2\ d/e = 1.7$. The fracture initiates through the leg of the welds at approximately 7000 psi at values less than 1.7. At greater values, the fracture initiates in the section of the undercuts of the plate at approximately 22,000 psi.

In the case of double fillets with V groove or those welded with rods of deep penetrations, the critical value of $2\ d/e$ is at 0.6.

Consequently, the use of V grooves or rods of deep penetration seems to be more important from the point of fatigue strength for double fillets with dimensions characterized by $0.6 < 2\ d/e < 1.7$.

D. Vasarhelyi, USA

4128. Habicht, F. R., Stresses in dowel connections (in German), *Planen u. Bauen* **4**, 12, 397–400, Dec. 1950.

Paper analyzes stresses in dowel connections when the material is supposed to be elastic. Influence of the elastic deformations of the dowels is considered. In the results, it is pointed out that the stresses grow due to these dowel deformations. It should, however, be of interest for the design if the plastic stages could also be investigated.

Henrik Nylander, Sweden

Structures

(See also Revs. 4119, 4156, 4330, 4332)

4129. Zaytzeff, S., The Cross method and its simplifications (in French), *Tech. mod. Constr.* **5**, 6, 165–169, June 1950.

4130. Michielsen, H. F., Calculation of deflections in indeterminate structures, *J. aero. Sci.* **18**, 4, 284–285, Apr. 1951.

Paper discusses form of the deflection equations set forth in certain papers by Boley and Moore and also by Cox and Sopworth. It contains a brief discussion of the advantages of using the equation in the form derived by Boley and Moore.

Nicholas Di Pinto, USA

4131. Fraeys de Veubeke, B., Diffusion of hyperstatic unknowns in wing units with coupled spars (in French), *Bull. Serv. tech. aéro.* no. 24, 57 pp., 1951.

Author studies the bending and twisting of an airplane wing consisting of two long caisson-type spars interconnected by a system of ribs which are perpendicular to the length of spar. He also presents some discussion of the case of diagonal ribs. The method of Friedrichs and von Kármán is developed and generalized by allowing for elasticity of the connecting ribs. The wing is conceived as being a statically indeterminate system of discrete members as well as approximated by a continuous medium with appropriate stiffness properties.

About one fourth of paper is devoted to an introductory review of the usual variational method as applied to continuous elastic media. Special attention is given to multiple connectivity and dislocations, so that the theory may be applicable to the given structural problem.

Sturm-Liouville theory is used for the case in which "perfect coupling" is assumed. Under this definition, the shearing deformations of spars are neglected and ribs are considered to be rigid. The standard Ritz method for obtaining stationary values of an integral by differentiating with respect to coefficients in an assumed infinite series is also applied in the case of perfect coupling. Finally, recurrence formulas required for the analysis of discrete systems which are statically indeterminate are explicitly developed, both in the case of ribs perpendicular to the spars and in the case of diagonal ribs.

Paper no doubt meets the requirements of its purpose. However, reviewer thinks it would be more valuable had author devoted more space to the detailed analysis and its results and less space to the formalities of standard elasticity theory.

W. H. Hoppmann, II, USA

4132. Roland, E., Calculation of continuous porticos of arbitrary form (in French), *Ossature métallique* **15**, 10, 11; 471–483, 536–545; Oct., Nov. 1950.

Author deals with continuous porticos built with vertical columns and beams of arbitrary shape between them. The steps in the calculation are: (1) All joints supposed rigidly fixed, couples M and thrusts Q are calculated. (2) Couples M and thrusts Q are distributed among the bars of each joint, as could be done in the Cross method. (3) Having obtained the couples and thrusts at each end of every bar, their influences are transmitted (carried over) from left to right and then from right to left over the entire structure, adding the influences in each end of each bar. In these operations the fundamental system is obtained, supposing all the beams with ends simply supported on columns and fulfilling, step by step, the condition of continuity of the structure (as in Gauss' algorithmus).

Considering that the unknown moments M form a system of

equations of three terms (Clapeyron's), and that the same occurs with the thrusts Q, the influences between them are taken into account in auxiliary systems of equations in each step [like (14) and (19)]. Ending this transmission of influences, final moments and thrusts are obtained by adding those of steps (2) and (3).

In reviewer's opinion: (a) The notation employed in this work is complicated and uncommon; (b) the difficulty in obtaining the constants of beams with very different shapes seems to imply that it would be more convenient to solve these structures directly with statical unknowns and the same fundamental system used in step (3); (c) reviewer does not dare to consider this a completely new method, as author states.

Arturo J. Bignoli, Argentina

4133. **Efsen, A., Statical computation of general Vierendeel frames,** *Bygnstat. Medd.* **22**, 1, 25 pp., 1951.

Presents a method for solution of plane Vierendeel frames which need not have any symmetry. All joint translations are prohibited by the substitution of fictitious diagonals for which imaginary internal forces are computed. The actual stresses are then determined by algebraically adding the stresses of the imaginary system to those obtained in removing the fictitious diagonals. Special cases are treated including the parallel post case, redundant supports, and systems with very large and very small stiffnesses. Numerical examples are worked. Reviewer found the paper difficult to read because of inadequate definition of terms and constants that were developed in earlier papers by the same author.

Morley English, USA

4134. **Yang, C. H., Beedle, L. S., and Johnston, B. G., Welded continuous frames and their components,** *Weld. Res. Suppl.* **16**, 7, 348s–356s, July 1951.

Paper gives numerical examples of practical design for fixed-ended beams using elastic analysis for working load and plastic analysis for ultimate load (working load multiplied by safety factor). Appreciable saving in material results when structure is elastically inefficient for given loading distribution, i.e., when first plastic hinge forms at load considerably below ultimate.

Deflection at working load for beam designed by plastic analysis falls between values obtained by elastic analysis for simply supported beam and fixed-ended beam.

Since plastic hinge method of Greenberg and Prager [AMR **3**, Rev. 2273] gives unlimited deflections at ultimate load, authors recommend a conservative modification for design practice since they believe the deflections at ultimate load, as well as at working load, should be limited by design specifications. Reviewer is not convinced that this conservatism is desirable.

Stanley U. Benscoter, USA

4135. **Stüssi, F., Steel hangars of large spans** (in Spanish), *Cons. Super. Invest. Cienc., Inst. tecn. Constr. Cem. Madrid* no. 98, 24 pp., 1951.

A formula is derived for estimating the weight of trusses, and curves are included for use with this formula in comparing two possible roof-framing plans for a hangar. Author recognizes that total cost, which includes cost of foundations, is the determining factor. This is of importance in comparing, for example, simple trusses and arches. Three roof-framing schemes for the Kloten hangar, Zurich airport, were studied with the aid of the formula. The final choice was a three-hinged, trussed arch with intermediate hinge placed at the two-tenths point of the span. It is said that the increase in weight of the superstructure, as compared to the conventional three-hinged arch, is more than compensated for by the savings possible in the foundations due to smaller horizontal thrust.

James P. Michalos, USA

4136. **Selberg, A., Dampening effect in suspension bridges,** *Publ. int. Assn. Bridge Struct. Engng.* **10**, 183–198, Nov. 1950.

Main interest in this article is the results of tests made upon relatively small suspension bridges (widths from 2.85 m to 6.6 m and spans from 70 m to 160 m) in Norway. The tests were made by arranging for a number of young men to jump in unison with the period of natural frequency of the span until sufficient amplitude had developed to permit measurement of the decay. Measurement was made of amplitude and of strain on a stiffening girder. It was found that the logarithmic decrement was nearly independent of the amplitude and that the torsional stiffness of the concrete floor slab is an important factor in preventing catastrophic oscillations in short-span bridges. Formulas are also given for determining natural frequencies.

L. E. Grinter, USA

⊗4137. **Federhofer, K., Dynamics of arched girders and circular rings (Dynamik des Bogenträgers und Kreisringes),** Wien, Springer-Verlag, 1950, 179 pp., 35 figs, 26 tables. $5.50.

Volume is an orderly compilation of writings and present information on subject. Usual assumptions of elasticity are made. Equations are developed in vector notation for bending, twisting, and deformation along axis of curved members. Equations for frequency are developed for the closed circular ring, fixed-ended circular arc beam, and hinged circular-arc beam. Since the so-called exact basic equations are very complicated, practical approximate expressions are developed by use of the Ritz method. Equations, tables, and curves are given in dimensionless terms to expedite solutions and extend their usefulness. Much of the book is devoted to double-symmetrical and quasi-warp-free cross sections. Influence on frequency of variation of cross section and deviation of axis from circular arc are studied. Some test results are given for comparison with theory. The influence of radial internal (and external) pressure on the frequency of circular-arc beams and circular rings is discussed. The book closes with a section on the frequency of a two-hinged arch with parabolic axis.

Book is orderly, compact, and contains considerable information on vibrations of curved members and circular rings, and should be of interest to aeronautical, electrical, mechanical, and structural engineers.

Robert B. B. Moorman, USA

4138 **Czulak, J., Circular arches in rectangular coordinates** (in Polish), *Arch. Mech. stos.* **2**, 4, 319–347, 1950.

In 1935, W. Wierzbicki presented a method of analyzing parabolic arches by using a rectangular system of coordinates. Author applies this method to circular arches and demonstrates simplifications of calculation as compared with analysis in polar coordinates. Influence of direct stress and shear on elastic deformations is neglected. The relationship between radius, span, and rise of arch is expressed by a polynomial, the parameters of which are established by method of least squares and tabulated for various ratios of rise and span. It is shown that deviations from results of strict analysis are negligible. A further simplification of calculations is obtained for flat arches by introducing another polynomial, and particular values of its two parameters are tabulated. Possible errors in hyperstatical values of arches are examined.

J. J. Polivka, USA

4139. **Whitney, C. S., and Anderson, B. G., Plain and reinforced concrete arches,** *J. Amer. Concr. Inst.* **22**, 9, 681–691, May 1951.

Using the methods of analysis of the ultimate strength of concrete arches as outlined in the report of Committee 312 [title source, Sept. 1940], authors propose design procedure and load

formulas to incorporate effects commonly ignored. Special attention is paid to the problem of considering equilibrium in the deflected state, including the question of stability in the plane of the arch rib. E. F. Masur, USA

4140. Mercy, G., Elastic stability of rectangular foundations (in French), *Ann. Inst. tech. Bât. Trav. publics (N.S.)* no. 190, 19 pp., May 1951.

The classical formulas of Boussinesq giving the stress produced by a load acting on a semi-infinite mass, completed by Froehlich's assumptions, are used to determine the elastic limit of the soil under a rectangular foundation. Considering the pressure on a very small area as a concentrated load and integrating the values over the whole rectangular surface of the foundation, the principal stresses are obtained. By means of Mohr's circle, the limit equilibrium is then established. Because of the complicated expressions given by the integration, tables and graphs are given for different proportions between the sides of the rectangle, for practical use. Aurel A. Beles, Rumania

4141. Koval'skiĭ, B. S., Bunkers with flexible walls (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **77**, 6, 981–983, Apr. 1951.

A theoretical analysis by membrane theory of shape of, and stresses in, suspended thin-walled bunkers. Fill is assumed with horizontal top surface so that vertical components of pressure are proportional to distance from that surface, and horizontal components are obtained from vertical by Rankine-Coulomb theory. Wall friction and possible silo effect are neglected, which may be misleading in deep and narrow bunkers. In view of membrane theory, influence of flexural wall rigidity is neglected, which may not be as small as usually assumed. Treatment is the most nearly complete and rigorous publication known to reviewer, though some private firms have worked out, but not published, similar solutions. Formulas for shape under load are particularly important practically, since differences in shape of loaded and unloaded bunkers are considerable and attachments, such as unloading devices, must be able to follow the resulting movements. George Winter, USA

4142. Hiltscher, R., and Elfman, S., Theoretical and photoelastic investigation of mountain tunnels (in Swedish), *Tekn. Tidskr.* **80**, 30, 727–730, Aug. 1950.

Problem investigated is stress distribution round a long tunnel at considerable depth. First part of paper shows that problem can be treated as plane and makes use of known solution for elliptical hole in infinite plate. Second part shows result from photoelastic investigation of different tunnel section shapes. Stresses determined for elliptical shape agree well with theory. Vaulted rectangular shapes with rounded corners are chosen so that stress in top of tunnel vault is zero, and this is shown to occur at a certain relation between the ratios total height to width and height of vault to width. Sverker Sjöström, Sweden

4143. Klouček, C. V., Structural analysis by distribution of deformation, *Quart. appl. Math.* **9**, 1, 77–88, Apr. 1951.

By considering an elastic beam-column structure and a four-span beam, author discusses theoretically the basic relations of his method of structural analysis by distribution of joint rotations. [Cf. author's recently published book "Distribution of deformation. A new method of structural analyses."] By following in detail the distribution of joint rotations as the joints of an elastic structure are locked and unlocked after the Hardy Cross and Grinter manner, it is shown that the distribution process develops in the form of geometric series. For any one joint the series can be summed and result written in finite form. Distribution factors thus become available by means of which the rotations of joints representing the final equilibrium configuration can be computed in one step. (Reviewer and others have devised similar procedures for direct distribution of moments, although not in such complete detail.) F. S. Shaw, Australia

4144. Amorena, L. G., Torsion in floor supporting beams and moment distribution in junction points in structures with rigidly connected floors (in Spanish), *Informes Construc.* **3**, 29, 422–428, 1–6, Mar. 1951.

Considering the floor composed of beams normal to the supported edge, with no action between them, author finds the expression of stiffness and carry-over factor of a joint formed with the beam at the edge (working in torsion) and the two adjacent floors (in bending). Knowing those values, evidently any method of deformations is applicable for the calculation of such a floor, particularly the Cross method. Such is the conclusion of the author, which in reviewer's opinion could be reached more quickly in a most general way.

Author says he does not know other studies on the same subject. The same particular case was studied in Spanish by C. Laucher [*Cienc. y Técn.* **106**, 528, p. 403, June 1948] including a numerical example. Arturo J. Bignoli, Argentina

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 4108, 4123, 4158, 4167, 4171, 4176, 4177, 4197, 4349)

4145. Schmid, E., and Boas, W., Plasticity of crystals, translation from "Kristallplastizitaet mit besonderer Berücksichtigung der Metalle," London, F. A. Hughes & Co., Ltd., 1950, xvi + 353 pp. £1.15.0

Book was published originally in German in 1935 based on lectures given in 1931. Despite subsequent great development in the field, no recent book can replace this in all respects. It is a classic and, as such, necessary in the library of every scientific worker in metal plasticity, if not on his desk. Most important newer developments are in "An introduction to the physics of metals and alloys" by Dr. W. Boas [Melbourne University Press] and in C. S. Barrett's "Structure of metals" [McGraw-Hill Book Co.]. The complex deformation behavior of magnesium and the importance of this metal in modern technology explains the interest of Magnesium Electron Limited in providing this translation, a tribute by industry to the practical importance of fundamental studies.

Specifically, the book deals with: I. Fundamentals of crystallography, including lattice structure, symmetry, representation of planes and directions, crystal projection, simple theorems. II. Elasticity of crystals including Hooke's law, simplification of Hooke's law as a consequence of crystal symmetry, Cauchy relations, determination of elastic parameters. III. Production of crystals by: recrystallization after critical plastic deformation, grain growth, crystallization in crucible, drawing from the melt, precipitation from vapor, electric deposition. IV. Determination of orientation of crystals by mechanical, optical, and x-ray methods. V. Geometry of mechanism of deformation: simple and double glide, mechanical twinning. VI. Plasticity and strength of metal crystals: Elements of gliding and twinning, dynamics of glide and twinning, fracture along crystallographic planes, aftereffect phenomena and cyclic stressing, physical and chemical changes of cold working, recrystallization. VII. Plasticity and strength of ionic crystals: gliding and twinning, fracture, Sohnke's normal stress law, effect of cold working and of solvent on mechanical properties, Joffé effect. VIII.

Theories of crystal plasticity and strength: theoretical tensile and shear strengths, discrepancy of technical values, work-hardening, recrystallization, atomic migration, plasticity. IX. Properties of polycrystalline technical materials in relation to the behavior of the single crystal. Hans F. Winterkorn, USA

4146. Churchman, A. T., and Cottrell, A. H., Yield phenomenon and twinning in α-iron, *Nature Lond.* **167,** 4258, 943–945, June 1951.

Whereas stress does not decrease markedly at the yield point in room-temperature tests of single crystals of iron containing small amounts of carbon or nitrogen, in contrast to behavior of polycrystalline iron, authors observe sharp drop in slightly carburized (0.02% carbon) single crystals of iron tested at −77 C. On further lowering of temperature to −190 C, yield phenomenon is replaced by deformation twinning; this is marked by a series of sudden decreases in stress accompanied by audible clicks. A thoroughly decarburized and denitrided crystal did not twin but underwent slip at −190 C. Twinning at −190 C is also inhibited in a carburized (0.02% carbon) crystal by immediate prior plastic strain through the yield point at room temperature. George V. Smith, USA

4147. Jenkins, W. D., Creep of high-purity aluminum, *J. Res. nat. Bur. Stands.* **46,** 4, 310–317, Apr. 1951.

Effect of various creep rates on deformation mechanism of cold-drawn aluminum (40% reduction of area) was studied at 105 F. Similar to chemical rate reactions, flow ability increases with increasing strain energy. Strain-hardening decreased monotonically with increase in strain during first stage. Increase in stress increases extension on loading and, at beginning of third stage, decreases time for initiation of each stage. Variation of stress with time to obtain equal strain values appears to follow power relationship. Cyclic temperature changes appear to increase ductility. Discontinuous flow is observed in constant-load creep tests at 105 F and in tensile tests at 80 F. Within limited range of strain rates in first stage, conformance with Andrade's law of transient flow is found. Stress-creep rate curves on log-log or semi-log basis are sigmoidal. Microstructure and fracture characteristics are discussed and made the basis for a proposed flow mechanism. Hans F. Winterkorn, USA

4148. Green, A. P., The use of plasticine models to simulate the plastic flow of metals, *Phil. Mag.* (7), **42,** 327, 365–373, Apr. 1951.

In the theory of the plasticity of metals, experiments are made both to provide facts for creating theories and to test the validity of such theories. This often requires elaborate and expensive apparatus and involves serious delay. Author considers to what extent the behavior of plasticine approaches that of the ideal metal. A stress-strain curve in compression is given which has an initial steep straight section bending over quite rapidly to a straight almost horizontal portion. Figures quoted indicate that creep is negligible and that plasticine is virtually incompressible.

The present paper deals with plane strain. The specimen is sectioned in a plane of symmetry and a square grid imprinted.

Examples given are: (1) Direct extrusion through smooth and rough square dies; (2) direct extrusion through a smooth wedge-shaped die; (3) indentation with a lubricated wedge; (4) indentation with a series of wedges; (5) compression of a wedge by a smooth flat die; (6) indentation of a deep narrow block by a smooth flat punch.

For examples 1, 2, and 3, the theoretical deformation of the grids is computed from slip-line fields given by Hill. The experimental deformations are in close agreement. It is noted that the velocity discontinuities in the theoretical solution for a non-hardening plastic-rigid material become regions of high shear rate in the actual work-hardening material. The assumed shape of the dead metal region in the theory in direct extrusion with rough walls is strikingly verified. Example 4 is interesting in that no complete theory is available.

The deformation of a plasticine and of an annealed copper wedge are shown with the author's theoretical solution. The indentation of a deep narrow block corresponds closely to an unpublished solution by Hill.

The results confirm that the plane deformation of plasticine under large strains approximates closely to that of an ideal metal. Comparison of theoretical and experimental results shows a striking correspondence. Reviewer believes that the technique is an important addition to the methods available to both experimental and theoretical plasticians. Proposed extension of the experiments to three dimensions and to quantitative measurements should provide much-needed information.
J. F. W. Bishop, England

4149. Schremp, F. W., Ferry, J. D., and Evans, W. W., Mechanical properties of substances of high molecular weight. IX. Non-Newtonian flow and stress relaxation in concentrated polyisobutylene and polystyrene solutions, *J. appl. Phys.* **22,** 6, 711–717, June 1951.

Viscosity of concentrated polymer solutions of polyisobutylene and polystyrene are measured in concentric cylinder apparatus. Rate of strain is found to be proportional to the hyperbolic sine of shearing stress.

Stress relaxation also is studied after steady-state flow was reached. Curves obtained are explained by assuming a distribution of relaxation times. A logarithmic distribution of elastic mechanisms relaxed by hyperbolic sine flow mechanisms enables the distribution function to be calculated. This function is independent of rate of shear. W. Lethersich, England

4150. Oldroyd, J. G., Rectilinear flow of non-Bingham plastic solids and non-Newtonian viscous liquids, II, *Proc. Camb. phil. Soc.* **47,** part 2, 410–418, Apr. 1951.

The flows indicated in the title are caused by the given motion of the boundaries in the absence of a pressure gradient. The basic equations are found to be closely analogous to those governing steady two-dimensional irrotational subsonic flows of a gas with an arbitrary pressure-density relation. This analogy suggests the use of a Legendre transformation to linearize the problem. The flow caused by the axial motion of an infinite elliptic cylinder in an infinite plastic solid is treated as an illustration of the method. W. Prager, USA

4151. Scheele, W., and Timm, Th., On the relationship between the temperature dependence of non-Newtonian flow in chain polymerizates and their freezing region (apparent second-order transition region) (in German), *Kolloid Z.* **121,** 3, 144–147, Mar. 1951.

Paper is extension of previous investigation by authors [*Kolloid Z.* **116,** p. 129, 1950] of flow properties of five polyvinyl acetates of varying degree of polymerization as function of temperature. Such flow is adequately described by the Waele-Ostwald equation, $v = K\tau^n$, where v is velocity of deformation, τ shear stress, K and n constants. Exponent n passes through maximum at temperature T_n which falls close to region of apparent second-order transition ET as determined dilatometrically. Authors suggest T_n may characterize transition region ("freezing range") equally well as ET. T_n is found to be a func-

tion of deforming stress, i.e., of observational time scale just as ET is function of rate of cooling in dilatation experiments.

Support of "isoviscosity" hypothesis of ET is advanced by showing work of deformation between constant limits at T_n to be virtually identical for a series of 17 polymers consisting of polyesters and polyamides.

J. T. Bergen, USA

4152. Calnan, E. A., and Clews, C. J. B., Deformation textures in face-centered cubic metals, *Phil. Mag.* (7), **41**, 322, 1085–1100, Nov. 1950.

A method for calculating the tensile stress-elongation curve for polycrystalline face-centered metals, based on the data of single-crystal test, is shown. The constraints imposed by the surrounding grains of the aggregate combining with applied stress give effective stress on the grain with direction deviated from the applied stress. This deviation in direction increases with the applied stress and keeps the shear stress on the slipping plane from reaching the critical value of slipping until the shear stress at the corner point of stereographic projection of unit triangle due to applied stress reaches this critical value. Then, multiple slipping is available and deformation can occur without misfit at the grain boundaries. It is assumed in multiple slipping, with equal shear stresses in different planes of slipping, that no rotation of crystal is produced. For grains with less lateral support from the surrounding grains, rotation will occur due to single or duplex slipping. Based on the above assumptions, calculation of a tensile stress-elongation curve is shown. It agrees with experimental data and that derived by Taylor. Deformation texture under tension, compression, rolling, and drawing, based on rotation due to single and duplex slip, are discussed.

T. H. Lin, USA

Failure, Mechanics of Solid State

4153. Bushmanov, E. L., Potak, Ya. M., and Sachkov, V. V., On the effect of alloying on resistance of iron to brittle failure (in Russian), *Zh. tekh. Fiz.* **21**, 1, 26–31, Jan. 1951.

Influence of alloying elements Ni, Cr, Si, Cu, Co, W, Mn and P in iron with less than 0.03% C is investigated. Elements are added in small amounts of 5% weight or less. With regard to resistance, tensile tests at liquid-nitrogen temperature show independence of composition at the same medium size of ferrite grains. After the same heat treatment, alloys differ in resistance only because of different grain size. Resistance decreases with increasing grain size; as, e.g., Ni hinders growth of grains and P favors it, Fe-alloy with 3% Ni is more ductile than iron containing 0.28% P. Experimental results are explained by theory of Laschko [*Zh. tekh. Fiz.* **18**, 7, 1948].

Heinrich Mussmann, Germany

4154. Regel', V. R., On the mechanism of brittle failure of plastics (in Russian), *Zh. tekh. Fiz.* **21**, 3, 289–303, Mar. 1951.

Observations on numerous tests of 5 sec to 20 days' duration are summarized to clarify the mechanism of brittle fracture of plastics, with particular reference to the Griffith-Joffe theory of progressive formation of microcracks. It is maintained that the present tests verify this theory not only in their over-all results (as has been done before with other materials) but by actual observation, visual and on reproduced microphotographs, of the successive crack formation and propagation. It is shown that for the given plastic, polymethylmethacrylate with 10% plasticizer, the rupture surface always consists of two zones, one glossy and generally near the surface, and the other dull. The glossy part is interpreted to represent the original, critical crack, forming at a surface defect. Microscopic observation of the dull portion reveals a regular and consistently repetitive pattern of hyperbolic, regularly oriented cracks. From the parameters of these hyperbolas, author draws quantitative conclusions regarding velocity of crack propagation. Statistics on number and size of cracks vs. time at a variety of stress levels are given and are said to confirm the statistical theory of brittle fracture. This fracture is said to occur in two stages: (1) The formation and growth of primary cracks, and (2) the considerably shorter period when overstressing of the ends of one of the primary cracks leads to rapid crack propagation and fracture. The second stage confirms existing theory. It is maintained, however, that the growth of primary cracks in the first stage takes place at maximum stresses (at ends of cracks) which are less than the theoretical, intermolecular strength. Author maintains that this discrepancy is explained, i.e., the necessary additional stress is supplied, by stresses caused by thermomolecular motion.

George Winter, USA

4155. Skidmore, W. E., Bursting tests of rotating disks typical of small gas turbine design, *Proc. Soc. exp. Stress Anal.* 8, 2, 29–48, 1951.

Data are presented on the permanent stretch at given speeds above the yield speed, and the burst speed, in a series of 14-ST aluminum and SAE 4130 steel disks. The effect of profile, centrally located holes, and blade loading are investigated. Comparison is made of the average and maximum elastic stress at the yield speed and burst speed with material properties determined from tensile test coupons.

A small central hole in a disk of ductile material was found to have a strength-decreasing effect in proportion to the section area of material removed rather than in accordance with the twofold increase in maximum elastic stress. All disks, regardless of shape, burst at an average elastic stress approximately 10% below the material tensile strength.

A poor correlation is shown to exist between the maximum elastic stress and material ultimate tensile strength in contoured disks designed according to the Stodola constant-stress disk formula, $y = y_a e^{-uv^2/2\sigma}$. Reviewer suggests that the discrepancy results from the fact that these disks were not subjected to radial rim loading (blade loading) of stress σ, which is necessary to make the disk of the designed profile truly a "constant stress" disk.

S. S. Manson, USA

Design Factors, Meaning of Material Tests

4156. Torroja, E., Conception and calculation of the factor of safety for structures in reinforced concrete (in French), *Ann. Inst. tech. Bât. Trav. publics* (*N.S.*) no. 194, 12 pp., June 1951.

Report is a thorough analysis of all factors involved in the conception of the factor of safety, and emphasizes that this factor must be related to the economy of structural unit considered and the loss likely to be incurred if failure takes place.

It is further suggested that the basis of strength calculation should be the stress exceeded by 5 out of 6 test specimens, rather than the mean ultimate stress of 6 specimens.

Report contains three appendixes. The first gives safety factors obtained by statistical analysis of actual structures; the second shows that theoretical values for S.F. proposed are close to present practice; the third stresses that it is necessary to specify scatter of sample test results as well as limiting value.

Appendixes seem to suffer from too much abbreviation, and logical sequence of steps is not apparent nor is it evident which points are based on statistical theory and which on practical or experimental results.

Frank A. Blakey, Australia

4157. Späth, W., The conduct and interpretation of static tests (in German), *Schweiz. Arch.* **17,** 6, 177–181, June 1951.

Author calls attention to the (generally recognized) arbitrariness in defining mechanical properties in terms of prescribed strains, e.g., by the "offset yield strength." He points out that equally valid definitions are possible in terms of the relaxation load required to return the material from a prescribed deformation to its initial state of deformation. He presents some experimental curves for relaxation load as a function of imposed load for a cantilever beam deflected by a transverse load at the free end. The paper concludes with a plea for defining yield strength in terms of the ratio of permanent to elastic deformation rather than in terms of an arbitrary offset value. Author is apparently unaware of the paper by W. R. Osgood [*J. appl. Mech.* **7,** A61–A62, 1940] in which the arguments in favor of this proposal were presented effectively more than ten years ago.

Walter Ramberg, USA

Material Test Techniques

(See also Revs. 4102, 4111, 4124, 4350)

4158. Clark, D. S., and Duwez, P. E., The influence of strain rate on some tensile properties of steel, *Proc. Amer. Soc. Test. Mat.* **50,** 560–575, 1950.

Testing equipment is described in which strain rates as high as 200/sec can be obtained in tubular specimens loaded by internal pressure. The pressure is produced by an ingenious loading cylinder and plunger. This cylinder is filled with mercury and the plunger is struck by the falling weight of a "sling-shot"-type machine. Authors state that this type of loading produces a more uniform strain rate than that of a tensile impact test.

From test specimens cut from plates of three types of steels, the following general results are reported. The ultimate strength increases with increasing strain rate. This increase is about 45% for carbon steel and about 30% for manganese and nickel-chromium steels. Above a rate of 100/sec the increase in the ultimate strength is small.

The proportional limit measured by the author is the stress corresponding to the force at which the recorded force-time oscillogram starts to curve. This proportional limit increases with rate of strain, and at rates greater than about 70/sec it is about equal to the ultimate strength.

Since the specimens were cut from plates, considerable scatter was obtained in the test results; the above conclusions are made from the average of several tests.

M. J. Manjoine, USA

4159. Werren, F., Mechanical properties of plastic laminates, *For. Prod. Lab. Rep.* 1820, 14 pp., 6 tables, 37 figs., Feb. 1951.

Results of constant-strain-rate tests in tension, compression, bending, and shear for 13 glass-fabric laminates and one cotton-fabric laminate are reported. Specimens were subjected to normal or to wet conditioning before conducting test. After exposure to wet atmospheric conditions, mechanical properties of laminates were substantially reduced. Directional property in tension and compression was appreciably large. Author tries to correlate the panel shear test data in considering the anisotropy of laminates.

Stress-strain curves and relations between tangent modulus and stress are shown for each laminate. For the latter, reviewer believes that it would be more valuable to plot relations between secant modulus and stress instead.

Test procedures are described in detail. The panel shear apparatus seems to be very interesting.

Ling-Wen Hu, USA

4160. Kjaer, V. A., Wearing test on materials for cylinder liners in marine engines, *Trans. Dan. Acad. tech. Sci.* no. 6, 56 pp., 1950.

A wear-testing machine is described which was developed for measuring the relative resistance to wear of materials for cylinder liners in marine engines. Great emphasis is laid on the importance of reproducing the actual wear conditions in engines as closely as possible in order that reliable ratings of wear resistance can be made. In the tests described, special efforts were made to obtain the same surface pressures and the same lubricating conditions and temperatures which exist in the types of engines of interest. The tests were successful in properly rating materials of known service performance. Paper also contains considerable general discussion of factors affecting wear in marine engines and of methods of calculating and expressing the amount of wear.

W. T. Lankford, Jr., USA

4161. Yustein, S. E., Winans, R. R., and Stark, H. J., Outdoor weather aging of plastics under various climatological conditions, *Amer. Soc. Test. Mat. Bull.* no. 173, 31–42, Apr. 1951.

The effects of outdoor weather aging under widely different climates are investigated for various types of plastic materials. The report covers exposures for 1, 3, 7, and 12 months. Subsequent reports will cover 18-, 24-, 30-, and 36-month exposures.

The materials dealt with in this report include 5 types of clear transparent sheet plastics, 6 types of laminated materials, and 5 types of molded terminal bars. The sheet materials are evaluated after each period of exposure for tensile and flexural properties, hardness, and dielectric constant and power factor. The electrical properties are determined for frequencies of 60, 1000, and 10^6 cycles. The transparent materials are evaluated also for light transmission and haze. The molded terminal bars are evaluated for insulation resistance, dielectric strength, and high-impact (HI) shock resistance.

On the basis of the extensive data accumulated at the completion of the first year's exposure, it is possible to deduce the occurrence of a variety of effects that appear to be related to differences in the climatic and environmental conditions and in the exposure periods.

From authors' summary

4162. Althof, F. C., Accelerated test for stress corrosion (in German), *Metall* **4,** 13/14, 267–273, July 1950.

4163. Moriarty, C. D., Ultrasonic flaw detection in pipes by means of shear waves, *Trans. Amer. Soc. mech. Engrs.* **73,** 3, 225–229, Apr. 1951.

The shear waves are introduced as pulses into the pipes. The normal angle-searching units of the reflectoscopes are adapted to this purpose by grinding the plexiglass piece to a curvature fitting the diameter of the pipe. The angle of incidence is 32°, the angle of shear waves on the material being 45°. The shear waves ricochet off the inner and outer pipe boundaries in their travel circumferentially and return to the searching unit. Pulse length must be made not too small. The sensitivity is controlled by artificial notches with a depth of 5% of wall thickness. These notches reflect the shear waves strongly. It is proposed that each pipe to be tested be given such a notch at one end. It is possible to distinguish the injurious from the noninjurious defects, such as die marks, tight scale breaks, etc. The limit is seen to be a depth of 5% wall thickness. The method is limited by geometry. The pipe must have 4-in. OD at least, when the first revolution of the wave is used. Taking into account also the second, third, or fourth revolution, smaller diameter may also be inspected (e.g., $1^3/_4$ in.). Wall thickness should not exceed 20% of the inner diameter. The inclination of the notches

influences the sensitivity of the method. In an appendix, author shows that pipe welds may be inspected, if it is possible to make the welding-surface as smooth as the pipe surface. A discussion shows that this method is used in technical practice.

O. Ruediger, Germany

4164. Berdan, D., and Bernhard, R. K., Pilot studies of soil density measurements by means of x-rays, *Proc. Amer. Soc. Test. Mat.* **50**, 1328–1338, 1950.

Paper describes a method of using x rays for determining the effect of soil densification on radiographs and the corresponding motion of differential pressure cells buried in the soil. Apparatus used is of industrial make. Characteristic prints are shown for two typical soils, and the technique seems to yield satisfactory results. Authors also suggest further development of the method, which might become useful for future application in the field.

From authors' summary by P. Wilh. Werner, Sweden

4165. Blevins, T. B., and DeFries, M. G., A nomogram for calculating the stiffness of elastomers, *Amer. Soc. Test. Mat. Bull.* no. 173, 59–60, April 1951.

Nomogram is shown for obtaining the stiffness in flexure E from tests with the Tour-Marshall-Olsen design apparatus. The latter is the same and the procedure is nearly the same as in ASTM Method D 747-48T. Authors, however, instead of actually plotting numerous readings and then fitting a straight line (D 747-48T), obtain the slope of the load-deflection diagram simply as an average from load readings for 10 and 15 deg (and presumably a zero reading). Elastomers of a wide range of stiffness were tested and E was calculated by the nomogram, ASTM D 747-48T, and Strechert [*ASTM Bull.* no. 157, p. 61, March 1949] methods; agreement is fairly good.

B. M. Axilrod, USA

4166. Rinehart, J. S., Some quantitative data bearing on the scabbing of metals under explosive attack, *J. appl. Phys.* **22**, 5, 555–560, May 1951.

Fracturing, or scabbing, of material near a free surface as the result of a compression wave may occur in driving piles or as the result of explosion.

A stress wave was set up by an explosive charge attached to the center of one face of a metal disk of steel, brass, copper, or aluminum alloy. Scabbing was observed for every material, provided that the disk specimen was thin enough. The fractures were brittle. A modified Hopkinson bar method was used for determining particle velocities of the stress wave, from which author computed stress at fracture surface.

Author's method of computation yields stress values about twice the correct value, granting that wave was purely elastic. Reviewer believes plastic waves may have occurred. Significance of results is obscured by the fact that stress waves were not plane but divergent.

B. G. Rightmire, USA

4167. Hartbower, C. E., and Pellini, W. S., Explosion bulge test studies of the deformation of weldments, *Weld. Res. Suppl.* **16**, 6, 307s–317s, June 1951.

Paper describes testing of welds in heavy plate under uniform loading secured by means of detonation of an explosive above the plate. Variations in stress fields imposed on the plates are obtained by modifying the geometry of the die cavity into which the plates are bulged. Photogrid-deformation studies after bulging indicate distribution of plastic strains. In a biaxial stress field, deconcentrations of plastic strain are observed transverse to the weld in cases where the yield strength of the weld metal appreciably exceeds that of the plate. In a strongly unbalanced biaxial stress field, it is possible to minimize nonuniformity of plastic strain by orienting the weld transverse to the direction of the principal load.

Authors point out, and reviewer believes it well to emphasize, that their test results are specific to conditions entailing plastic deformation of weldments. In the elastic range the elastic strains of all components of the weldments are determined by the elastic modulus, which is essentially invariant for plate material and weld metal. Accordingly, no modification of elastic strains should be expected, and the weldment should strain as if it were metallurgically homogeneous, regardless of differences in the yield strengths of the components.

Frederick J. Winsor, USA

4168. Schmidt, E., Experiments with the new concrete-testing hammer for determination of concrete quality (in German), *Schweiz. Arch.* **17**, 5, 139–143, May 1951.

Author describes hammer used for nondestructive testing of concrete. The hammer test uses a principle similar to that used in the scleroscope hardness test. Test results are analyzed statistically and correlated with compression tests. Care must be taken to adhere to strict test rules and to use a sufficiently large number of readings.

Andrew Brodsky, USA

4169. Herzig, E., Experiments with the new concrete-testing hammer in the Division for Concrete and Reinforced Concrete of the Federal Material-Testing and Experiment Station, Zurich (in German), *Schweiz. Arch.* **17**, 5, 144–146, May 1951.

Test procedure is described in greater detail than in preceding review. Limited tests to show the effect of aggregate size, age, form work, and curing are discussed. It appears that compression tests should be made to arrive at new correlation curves when the type of aggregate is being changed.

Andrew Brodsky, USA

Mechanical Properties of Specific Materials

(See also Revs. 4099, 4146, 4147, 4148, 4149, 4154, 4157, 4158, 4159, 4161, 4165, 4167, 4168, 4169, 4184, 4194)

4170. Chalmers, B., The structure and mechanical properties of metals, New York, John Wiley & Sons, Inc., 1951, ix + 132 pp. $3.50.

This is the second volume in a series of "Monographs on Metallic Materials" published by the Royal Aeronautical Society [see AMR **4**, Rev. 262 for Vol. I]. Author presents his subject simply and concisely in three parts. Chapter I, entitled "The structure of pure metals," contains discussions of crystal-lattice arrangement, growth, orientation, and boundaries. Purely physical explanations are offered, mathematics being purposely avoided for simplicity's sake. This is true of the entire book. In chapter II, author logically extends his discussion to alloys. Explanations are centered about the equilibrium diagrams of the chosen examples, the basic structural quantities (described previously) always being brought to mind.

Pointing out that the alloy rarely exists in its equilibrium state, the second part of the book brings out the structural modifications induced by mechanical deformation and heat treatment. Chapter III treats the former in terms of slip in the crystal. Both the single crystal and polycrystalline cases are discussed. Chapter IV on heat treatment describes the effects of temperature change on structure. Major portion of this chapter is devoted to the phases of medium carbon steel and their transformations. Chapter V offers a short discussion on the basic techniques used in determining the structure of an alloy.

It is well recognized that the mechanical properties of an alloy depend on the structural make-up of that alloy. The author treats this subject in the final part of the book (chapter VI). The properties are discussed in the same order as their governing regions appear on the stress-strain diagram. Some space is also devoted to creep and fatigue.

Book has merit for those seeking limited fundamental knowledge of this broad subject. Reading it requires only a meager background of elementary physics and chemistry. As author points out, book is not intended to be a reference source. Significant technical data are scant. There is no formal bibliography but, in the author's acknowledgments, reference is made to some of the material sources. Author mentions other texts that are more suitable for a starting point in a detailed study of the subject. Julius Miklowitz, USA

4171. Böklen, R., Some observations on steels in pure static bending (in German), *Z. Metallk.* **42,** 6, 170–174, June 1951.

Static bending tests of annealed bars of two steels (I. carbon steel St C10; II. chromium-molybdenum steel Cr-Mo St 1452.5) were observed by x-ray stress measurement, extensometers, and optical examination of polished surfaces. For steel I, all methods indicated plastic flow on the face in tension before the face in compression. The flow lines on both faces were wavy lines parallel to the axis of bending. For steel II, all methods indicated that plastic flow set in simultaneously on the two faces. The flow lines on the stretched face were parallel to the axis of bending; those on the compressed face were oblique to axis of bending. Further bending produced lines at 45° to the axis on both faces. Strain markings on the stretched side are depressions; those on the compressed side, elevations. Mechanisms are suggested for the effects observed. F. R. N. Nabarro, England

4172. Bonnell, D. G. R., and Harper, F. C., The thermal expansion of concrete, *Nat. Build. Studies, Dept. sci. industr. Res., London, tech. Pap.* no. 7, 22 pp., 1951.

Paper reports measurement of the coefficient of linear thermal expansion of concrete made with various aggregates and cements commonly used in Great Britian.

The values used in the design of structures that have become somewhat standardized in different countries are presented. Authors include extensive data to show the effects of mix proportions, age of concrete, conditions of curing, wetting and drying after curing, and types of cement on the thermal expansion of concrete made with gravel, stone and slag.

The thermal movement of the concrete specimen is measured over the temperature range of 32 F to 104 F by means of roller-type extensometers. The coefficient of thermal expansion of concretes was found to depend largely on the type of aggregate. Average values ranged from 3.4×10^{-6} per °F for concrete made with limestone, to 7.3×10^{-6} per °F for concrete made with siliceous gravel. J. Neils Thompson, USA

4173. Fukada, E., The vibrational properties of wood. I, *J. phys. Soc. Japan* **5,** 5, 321–327, Sept.-Oct. 1950.

Logarithmic decrements are recorded on the basis of electric charges applied, and Young's moduli are measured on the basis of resonant frequency of lateral longitudinal vibration of $30 \times 2 \times 0.5$-cm specimens representing 18 softwood and 10 hardwood species. The experimental data were obtained for the frequency range of 100 to 5000 cps at 10 C and 15% moisture content of the samples. While Young's modulus shows hardly any variation with change in frequency of vibration, the decrement varies considerably. Thus, the decrement increases gradually with frequency of softwood specimens and decreases with a maximum value within the test range of hardwood specimens. This definite variation of the logarithmic decrement is considered to be a characteristic behavior of the two basic wood types.

According to a few additional exploratory tests, the modulus of elasticity perpendicular to the grain is about one tenth, and the logarithmic decrement is approximately three times that for wood parallel to grain. The influences of both moisture content and temperature of wood on the vibrational properties will be reported on in a second paper. E. G. Stern, USA

4174. Johnson, E. E., Bending, compression, and shear tests of three samples of flight deck planking, *David W. Taylor Mod. Basin Rep.* 736, 4 pp., Oct. 1950.

Data were collected from laminated Douglas fir beams 3 in. and 4 in. deep and one laminated Douglas fir beam containing upper surface of maple-impregnated plywood. 40-in. beams were loaded at third points in accordance with existing specifications. Results include data from 29 SR-4 strain gages on each beam.

Observations relative to the technique were: (a) Transverse strains are exceedingly sensitive to transverse load distribution, i.e., unevenness of surface under load precluded possibility of observing Poisson's ratio effect. (b) Tight-fitting aluminum plates on ends of compression specimens prevented characteristic brooming type of failure at ends. (c) Use of thin coat of plaster of Paris between specimen and head of machine to provide uniform loading does not serve purpose well, due to uneven crumbling of the specimen.

EI's computed by vibratory testing were higher than the same computed from bending data by flexure formula. Discrepancy is ascribed to retarded elastic response effect in longer test. No other mechanical properties' computations other than unit-shearing strengths are presented. Paper contains neither conclusions nor discussion of results. Reviewer believes some interesting information could have resulted from interpretation of detailed data presented. Ben S. Bryant, USA

4175. Stillinger, J. R., Some strength and related properties of old-growth Douglas fir decayed by fomes pini, *Amer. Soc. Test. Mat. Bull.* no. 173, 52–58, Apr. 1951.

To determine specific usability of old-growth Douglas fir decayed by red ring rot *Fomes pini*, the wood was classified into three groups according to visual separation of decay stages. Physical and static strength properties and nail-withdrawal resistance were observed for the three groups of decayed wood.

In comparison with sound wood, average properties of decayed wood ranged from 74 to 101% for specific gravity, 43 to 88% for modulus of rupture, 42 to 90% for stress at proportional limit in flexure, 55 to 103% for Young's modulus of elasticity, and 36 to 81% for crushing strength parallel to grain.

Nail-withdrawal resistance was determined according to 12 to 15 tests per variable, at a pulling rate of 0.5 in. per min, on 8d plain-finished, cement-coated, and galvanized common, box, and "ring" nails immediately after nail driving and after drying and wetting of random-selected nailed wood samples. Since the design of the rings influences the nail-holding power, it is disappointing that a detailed or photographic description of the tested nail was not included in the published paper.

In comparison with the withdrawal resistance on the ring nail driven into wood when green and at 12% moisture content, on the average the withdrawal resistance was reduced 16% for the galvanized nail, 26% for the cement-coated nail, 31% for the common nail, and 46% for the box nail. During seasoning of the wood from green to 12%, 12 to 6%, and 12 to 6 to 16% moisture content, the average loss in nail-holding power was 27% for the ring nail, 32% for the galvanized nail, 38% for the box nail, 41%

for the common nail, and 57% for the cement-coated nail. These average data verify the advantage of using ring nails and the disadvantage of using cement-coated nails under given conditions.

Author concludes that present stress grades for Douglas fir could include more of wood somewhat decayed with *Fomes pini.* Also, the selection of appropriate nail types would allow use of large quantities of such wood for sheathing, roof boards, and subflooring in certain types of structures and, possibly, for core material for plywood. E. G. Stern, USA

4176. Blizard, R. B., Visco-elasticity of rubber, *J. appl. Phys.* **22,** 6, 730–735, June 1951.

A theory for the frequency dependence of the dynamic modulus and internal friction coefficient in rubber is developed on the basis of a simple physical model. The basis of the model is that usually employed in the kinetic theory of rubberlike elasticity, but for each rubber chain the internal friction is introduced by postulating viscous couplings between the chain and the average surrounding medium. The complex modulus for such chains with either free or cross-linked ends is calculated by employing an electrical analog. The complex modulus of the rubber is then obtained from those of the individual chains and the distribution function for chain length and type of termination. The results of the theory are compared with experimental results for various rubbers. However, the basis on which the comparison is made is not sufficiently clear to the reviewer to allow any assessment of the significance of the agreement obtained.

R. S. Rivlin, England

4177. Cottrell, A. H., and Bilby, B. A., A mechanism for the growth of deformation twins in crystals, *Phil. Mag.* (7) **42,** 329, 573–581, June 1951.

The Frank-Read theory [*Phys. Rev.* **79,** 4, p. 722, Aug. 15, 1950] of slip-band formation is extended to the case of twinning. Twinning is produced by rotation of an imperfect dislocation (twinning dislocation) around a pole dislocation having a screw component. In body-centered cubic lattice, the unit dislocation $\frac{1}{2}\,a[111]$ dissociates into a sessile pole dislocation $\frac{1}{3}\,a[112]$ and a twinning dislocation $\frac{1}{6}\,a[11\bar{1}]$, all lying in plane (112). Twinning dislocation can rotate into pure screw form, and then move in plane $(\bar{1}21)$. Movement in this plane generates stacking fault, and rotation around sessile pole $\frac{1}{3}\,a[112]$ lifts twinning dislocation so that successive revolutions generate stacking faults between successive pairs of $(\bar{1}21)$ planes, leading to twinning.

A similar mechanism cannot be devised for the face-centered cubic structure in agreement with the absence of mechanical twinning in this structure. F. R. N. Nabarro, England

4178. Vanderbeck, R. W., and Gensamer, M., Evaluating notch toughness, *Weld. Res. Suppl.* **15,** 1, 37s–48s, Jan. 1950.

4179. Stockdale, G. F., Tooley, F. V., and Ying, C. W., Changes in the tensile strength of glass caused by water immersion treatment, *J. Amer. ceram. Soc.* **34,** 4, 116–121, Apr. 1951.

Soda-lime, lead, and borosilicate glass rods were drawn into fibers (0.011-in. diam) and tested in tension in air and water after varying periods of treatment in desiccators and hot distilled water (90 C). Four groups of specimens were tested: (1) Control specimens; (2) untreated specimens broken under water; (3) specimens soaked in water at 90 C for various periods of time, then tested in air; (4) specimens soaked in water and tested in water. All three glasses showed a loss in tensile strength of about 12% when tested in water without any previous treatment other than fabrication and annealing. Prolonged soaking in water, however, produced an increase in strength. Soda-lime and lead glass showed an increase of about 20–25% and appeared to reach this maximum after a 4-hr soaking, whether tested in air or water. Borosilicate glass, after a 1000-hr soaking, showed a tensile strength increase of 100% with no indication of having reached a maximum. For each environmental condition, 144 specimens (half control and half conditioned) were made and tested. The data show a coefficient of variation of about 11%, which speaks well for the authors' techniques.

R. H. Carey, USA

4180. Gibbs, P., and Cutler, I. B., On the fracture of glass which is subjected to slowly increasing stress, *J. Amer. ceram. Soc.* **34,** 7, 200–206, July 1951.

A single number called "strength" does not serve to describe adequately the fracture properties of glass. The surface where the fracture originates may be represented by a continuum over which flaws, differing in nominal breaking stresses, are distributed. Paper represents a statistical study of the distribution of such flaws over the surface of plate glass, using data of fracture diameters obtained by pressing balls of steel or sintered carbide, $^1/_{16}$ to $^3/_8$-in. diam, against the glass under constantly increasing pressures.

Analysis is based on hypothesis that fracture is a rate process, involving the successive separation of interatomic bonds. Rate of crack propagation becomes a function of activation energy, applied stress, and bond-energy fluctuations. From these considerations, a qualitative curve has been estimated for the relationship between stress and rate of crack propagation. This indicates that a minimum stress is required for the initiation of crack propagation, and a rapidly increasing rate at high stresses. Reviewer notes that the general form of this curve is in accord with available information on this subject.

The distribution function for the flaws is assumed to be of the general form $S\sigma^b$, where σ is stress, and S and b are numbers which characterize the nominal strength of the glass. A probability function, a ratio of fractures in an incremental area beyond the contact circle to total fractures, when plotted against area, indicate a value of b in various tests between 2.05 to 5.7. From other computations, S was found to have a magnitude of the order of 3×10^{-4}.

Paper supports the flaw theory of the strength of glass, and the concept of fracture as a rate process. Information obtained is mainly qualitative, and the constants evaluated will depend greatly upon the surface conditions of the particular piece of glass under test. Reviewer notes that answers to the vexing problems of the numerical relationships between actual flaw dimensions and nominal stress, and between stress and propagation velocity of the fracture, are not attempted. He hopes that the rate-process approach will be carried further in this direction.

E. B. Shand, USA

4181. Peter, W., The influence of the surface condition of the workpiece on the cooling reaction in liquid quenching media (in German), *Arch. Eisenhüttw.* **21,** 11/12, 395–402, Nov./Dec. 1950.

Author reports on investigation of the influence of scale and salt layers on metal quenched in different media: a fused salt, a highly concentrated salt solution, oil, and distilled water. A hardenability investigation was also carried out in similar quenching media with three plain carbon steels. The significance of a curve is also pointed out which shows cooling velocity vs. temperature for an oil-quenched silver ball. Small balls of iron containing tiny thermocouples were used for cooling-velocity determinations. Bright metal surfaces on the balls were obtained

by the use of hydrogen atmospheres. A scaled surface was achieved by heating in air, and a salt layer was obtained by using a salt bath. With scale or salt layers present, the temperature range of most effective cooling was found to be spread out when quenches were used in which evaporation of the quench plays an effective part in cooling. In quenching media of this type it was also noted that a shift may occur in the temperature at which maximum cooling velocities are attained.

Carl A. Keyser, USA

Mechanics of Forming and Cutting

4182. **Chao, B. T., and Trigger, K. J., Cutting temperatures and metal-cutting phenomena,** *Trans. Amer. Soc. mech. Engrs.* **73**, 6, 777–787, Aug. 1950.

Comparison is made between conventional turning and orthogonal cutting with carbides and a continuous chip. Influence of tool-chip contact area on tool-chip interface temperature is discussed. Schematic diagrams depicting the reported interrelation of several metal-cutting factors are presented.

Interface temperature (called cutting temperature) was measured by Herbert-Gottwein tool-work thermocouple technique. Tangential and thrust tool forces were measured through minute deflections by the GE master electro-limit gage heads. Theory of mechanics of chip formation was used in obtaining results.

Partial results in first series of conventional turning and orthogonal cutting tests, done dry with carbides, on NE 9445 at 183 Bhn were as follows: 1. Increase in chip-thickness-ratio is accompanied by decrease in tool-chip contact area. 2. Increase in interface temperature is accompanied by decrease in tool-chip contact area and coefficient of friction.

Partial results in second series of conventional turning tests, done dry with carbides, on NE 9445 at 283, 311, 352, and 401 Bhn were as follows: 1. For quenched and tempered steels coefficient of friction is practically constant at different hardnesses.

Results common to first and second series of tests were as follows: 1. Increase in cutting speed, up to 500 fpm, is accompanied by increase in interface temperature and chip-thickness-ratio and by decrease in tangential force, thrust force, and tool-chip contact area. 2. Increase in work-material hardness is accompanied by increase in sliding stress (unit frictional force) and chip-thickness-ratio and by decrease in cutting forces, tool-chip contact area, and coefficient of friction.

Reviewer, in many milling tests, has found that tangential cutting force does not vary appreciably with changes in cutting speed up to 1200 fpm, and that increased work-material hardness entails higher tool forces, contrary to authors' findings in turning.

Paper is a welcome addition to literature and of great interest, as attested by discussions of W. W. Gilbert, M. E. Merchant and E. J. Krabacher, R. S. Hahn, A. O. Schmidt, and M. C. Shaw, all investigators in the same field.

A. O. Schmidt, USA

4183. **Tlusty, J., Forces in a hobbing machine,** *Engng. Rev. Prague* no. 6, 14–23, Dec. 1950.

Exact shape of the chips removed by a 207-mm diam hobbing cutter of 9 flutes and 32 teeth during the cutting of a 14-teeth and 18-mm module pinion is investigated. The total chip section removed by the cutter at any instant, average cutting force vs. average chip section, carriage idling-feed force vs. feed speed, vertical feed-drive torque vs. cutting resistance force, cutting and dividing worm output, drive losses and efficiencies in relation to machine output, effective cutting power (6 to 7 KW) at optimum drive efficiency, wear of cutter, and the deformations (rigidity) of key design parts of a hobbing machine are very effectively determined.

Reviewer believes these findings will be very valuable in designing a hobbing machine to meet the desired accuracy of the geometrical shapes it produces, in establishing the optimum cutter feed and speed, and in understanding the exact action of each tooth of the hobbing cutter.

Dimitri Kececioglu, USA

4184. **Finston, M., Thermal effects in calendering viscous fluids,** *J. appl. Mech.* **18**, 1, 12–18, Mar. 1951.

Calendering of some plastic materials, when conducted at high speeds, produces blistering. The problem is analyzed assuming viscous flow of the material as it passes between the rolls. The dissipation of energy in the viscous flow is considered to determine the effect on the temperature of the plastic material. By this analysis two layers with relatively high temperature, one under each surface, are found in the regions where blisters tend to occur. The results appear to account satisfactorily for the observed blistering.

William Schroeder, USA

4185. **Hessenberg, W. C. F., and Sims, R. B., The effect of tension on torque and roll force in cold strip rolling,** *J. Iron Steel Inst. Lond.* **168**, part 2, 155–164, June 1951.

In cold-rolling of metal strip, back-tension increases torque, and front-tension decreases torque. Both reduce roll-separating force and load on bearings. Total rolling energy is essentially unaffected by strip tension. Paper describes experimental results and presents simple approximate formulas. Authors conclude that more exact mathematical treatments are not justified.

Turner Alfrey, Jr., USA

4186. **Dugard Showell, D. W., The hot working of tin bronzes,** *J. Inst. Metals* **75**, part 5, 527–540, 1950.

4187. **Back, L. H., The hot working of lead and lead-rich alloys,** *J. Inst. Metals* **76**, part 5, 541–556, 1950.

4188. **Cooke, E. A., Form grinding by mechanical and optical methods,** *Instn. mech. Engrs. Indust. Admin. Engng. Production* **163** (*W.E.P.* no. 61), 265–278, 1950.

Paper presents aspects of design peculiar to form-grinding machines, particularly the copying and generating mechanisms; also indicates development trend of this class of design. Kinematical principles of copying mechanisms using pantograph and duplex parallelogram are analyzed; also principles of circular arc generating by rotation or translation. Examples are given of methods by which these principles have been applied in existing machines. Their application to a new design of machine to provide great versatility is described in considerable detail.

M. Eugene Merchant, USA

Hydraulics; Cavitation; Transport

(See also Rev. 4313)

4189. **Shal'nev, K. K., Cavitation of rough surfaces** (in Russian), *Zh. tekh. Fiz.* **21**, 2, 206–220, Feb. 1951.

On uneven surfaces of hydraulic machines, caverns sometimes appear arranged in the direction of the absolute or relative velocity on fixed or moving surfaces, respectively. Experimental investigations of the flow of water through rectangular gaps with one or more forms of unevennesses (triangular, bump in the form of a segment of circle, sharp edge) were undertaken and the results therefore may be applied mainly to the cavitation between the blades and the casing of the machine. The heights of the unevennesses varied between 0.12 and 0.24 in. The different

phases of cavitation were photographed with a high-speed camera and the cavitation noise was partly registered by a piezoelectric pick-up with amplifier and cathode-ray tube. The pressures at several points of the channel, the mass flow of water and its temperature were measured. In the first phase, the cavitation caverns originate periodically at a distance of 2.5 to 2.75a (a height of unevenness) downstream of the bump with a frequency from 100 to 300 per second. The "Strouhal number" $(Na)/v$ (N frequency, v velocity in the reduced section) varied from 0.10 to 0.23. The calculated results, based on some simplifying assumptions and using known hydraulic coefficients, give for the critical cavitation coefficient of the gap $\lambda = (p - p_v)/q$ (p, q static and dynamic pressure in the gap, p_v vapor pressure of the water) at which cavitation begins, values which are about 15% below the experimental results. The danger of cavitation in gaps depends on the relative height and the shape of the unevennesses and on the thickness and character of the boundary layer. When the relative height of the unevenness is very small the influence of its height and shape becomes negligible.

Anton Kuhelj, Yugoslavia

4190. Crump, S. F., Determination of critical pressures for the inception of cavitation in fresh and sea water as influenced by air content of the water, *David W. Taylor Mod. Basin Rep.* 575, NS 713-065, 37 pp., Oct. 1949.

Tests were made with fresh water and sea water in a portable water tunnel having a Venturi nozzle working section (2-in. and 0.670-in. diam) to determine pressure in the throat for inception of cavitation. Pressures were not measured at the throat but at points upstream and downstream from it. Throat-pressure determination was based on the assumption that, when the cavitation zone extends beyond the first pressure tap downstream of the throat, the pressure at that tap is the same as at the throat. Reviewer doubts the validity of this assumption, and author states in a footnote that in subsequent tests, in which pressures were measured at the throat, cavitation always started when the pressure was approximately equal to vapor pressure, regardless of the air content of the water.

A distinction is made between cavitation consisting of individual bubbles and steady-state cavitation in which there is a zone of separation. These tests show that with water that is saturated with air, either fresh or sea water, bubble cavitation starts when the absolute pressure is several feet of water above vapor pressure, whereas steady-state cavitation begins at approximately vapor pressure. Tests with deaerated water were made with fresh water only, and the pressure for inception of cavitation was found to diminish with the air content. For an air content of 60% of saturation, bubble cavitation starts at 5 ft of water (±2) below vapor pressure, while steady-state cavitation occurrs at −17 ft (±1).

Joseph Levy, USA

4191. Warren, J., A study of head loss in Venturi-meter diffuser sections, *Trans. Amer. Soc. mech. Engrs.* **73**, 4, 399–402, May 1951.

Study sponsored by Builders Providence and carried out by author in an excellent manner, produced interesting results disagreeing in part with earlier work by Gibson and Beckman. Minimum percentage of head loss for recovery section depends upon cone angle and diameter ratio β (on 6-in. size, lowest head loss observed was 8.9% for β = 0.50, and total cone angle 8°). Truncating of cones was not found to give improvement as indicated by Beckman, but gave a decrease or increase in loss, with larger cone angles showing tendency of decrease. For a certain length of recovery cones including truncated cones, there is a certain cone angle that will give the minimum percentage of loss. Among the curved-wall diffusers tested, those with a uniform rate of velocity head decrease gave the minimum loss. Advantage of smooth-surfaced rolled-steel cones over unmachined cast Meehanite cones was slight. Reviewer believes that author has made a valuable contribution toward minimum loss design of Venturi meters.

O. E. Teichmann, USA

4192. Szebehely, V. G., Relation between gas evolution and physical properties of liquids, *J. appl. Phys.* **22**, 5, 627–628, May 1951.

Four heavy lubricating oils, three light lubricating oils and three aircraft engine fuels were tested for viscosity, surface tension, and density, and a relation is established between the time rate of evolution and solution of air and the viscosity of the liquids. A definite relation was also found between solubility constants and half lives as well as between half lives of evolution and solution.

From author's summary by E. Mühlemann, Switzerland

4193. Richter, W., Solution of a flow problem in multi-connected pipes and channels (in German), *Ing.-Arch.* **19**, 2, 143–153, 1951.

The flow in a particular hydraulic network (triply indeterminate) is solved by a trial-and-error graphical process for the passive system alone and for two active systems obtained by introducing pumps into the original network.

Stephen H. Crandall, USA

4194. Baccaredda, M., and Baldacci, R., Surface tension and parachor in some branched and unbranched high molecular weight hydrocarbons, *Ric. sci.* **20**, 12, 1817–1819, Dec. 1950.

First results are presented to find if parachors are suitable for determination of amount of branching in high molecular weight polymers, the method being intended to replace the one involving the measurement of ultrasonic molecular velocity. On the basis of experimental results listed by authors, it is found that the parachor method is suitable for investigating the amount of branching in liquid or molten polymers, or nearly ideal solutions of polymers. It is also believed that the parachor method is easier to use and more widely applicable.

Bruno W. Augenstein, USA

4195. Pound, G. M., and LaMer, V. K., Surface tension of small droplets as a function of size from critical supersaturation data, *J. chem. Phys.* **19**, 4, 506–507, Apr. 1951.

Surface-tension values have been determined from the size of the stable nucleus of (liquid) droplets in terms of the critical supersaturation ratio required for their spontaneous production. These surface-tension values seem to deviate from the extension of the curve obtained from bulk phase surface-tension measurements at temperatures below the freezing point, for water between 220 and 280 K. The need for additional measurements is pointed out.

H. A. Einstein, USA

4196. Keyes, F. G., A summary of viscosity and heat-conduction data for He, A, H_2, O_2, N_2, CO, CO_2, H_2O, and air, *Trans. Amer. Soc. mech. Engrs.* **73**, 5, 589–595, July 1951.

The Sutherland equation fails to represent the viscosity of gases over wide temperature ranges. Another two-constant formulation based on the Lennard-Jones potential is not quite adequate, at least in the case of carbon dioxide (maximum deviation 3% in range 273–1500 K). Author proposes a three-constant, empirical equation satisfactory for all available experimental data on the gases listed in the title, except helium and argon, for which different sets of constants are needed in different temperature ranges.

Constants are given for the gases listed in the title. A theoretical formula given by Enskog for the viscosity of binary mixtures is tested for two mixtures and found satisfactory. The form of equation used for viscosity is satisfactory also for heat conductivity of gases, and a comparison with experimental data for carbon dioxide is given. Serge Gratch, USA

4197. Jobling, A., and Lawrence, A. S. C., Viscosities of liquids at constant volume, *Proc. roy. Soc. Lond. Ser. A*, **206**, 1085, 257–274, Apr. 1951.

Direct measurements of the variation of the viscosity of benzene and carbon tetrachloride with temperature at constant volume were carried out in a falling cylinder viscometer up to pressures of 3000 atmospheres. The results confirm the prediction of Andrade, and Glasstone and Laidler, that log η is proportional to $1/T$, and agree with recent measurements at very low temperatures (Giller and Drickamer) and very high pressures (Bridgman). The slope of the straight line is the energy of viscous flow activation which varies as the inverse square free volume $(v - b)$. The additive constant also seems to vary with the volume. Leon Trilling, USA

4198. Macagno, E. O., Mechanical analogy, and its application to hydraulics (in Spanish), *Cienc. y Técn.* **116**, 585, 91–120, Mar. 1951.

A thorough review of the method of mechanical similarity with application to the design of model experiments. In this method the relation between a model and its prototype is expressed in terms of a set of constant scale ratios for length, mass, and time. Also the ratios of the forces acting in the model are assumed to be the same as for the prototype. The method is applied to hydraulic systems in which gravitational, viscous, surface-tension, and elastic forces are supposed to be acting alone and in various combinations. It is emphasized that exact dynamic modeling is almost impossible, but that nevertheless it may be possible from model tests to determine the behavior of the prototype with sufficient accuracy for engineering purposes.

Louis Landweber, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 4195, 4247, 4255, 4257)

4199. Dolapchiev, B., General method for determining the stability of arbitrarily situated vortex streets (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **77**, 6, 985–988, Apr. 1951.

Author derives in a simple and very general manner the necessary conditions for the equilibrium of a vortex street composed of equally spaced point vortexes in two parallel and arbitrarily situated rows. These conditions permit one to solve the problem of stability or instability in each of the three possible cases: (1) symmetric, (2) alternating, and (3) asymmetric vortex streets. The method used is that of Kochin [*Dokladi Akad. Nauk SSSR* **24**, 18–22, 1939 = *Sobranie sochinenii* **2**, 479–485, Izd. Akad. Nauk SSSR, Moscow-Leningrad, 1949; or Kochin, Kibel' and Roze, AMR **3**, Rev. 510] for small finite displacements of vortexes as applied in cases (1) and (2), and, on the other hand, that of solution of the equations for the perturbed motion of the vortexes as outlined by the author in an earlier paper [*Z. angew. Math. Mech.* **17**, p. 313, 1937]. Author shows that the most general necessary condition for stability of a vortex street is the fulfillment of the relation (*) $\sinh \kappa\pi = \sin \lambda\pi$, obtained first by a different method by the author [*Godishnik Sof. Univ.* **39**, 287, 1942], where $\kappa = h/l$, $\lambda = d/l$, h the distance between the rows, l spacing of the vortexes in each row, and d the shift of one row with respect to the other from the symmetrical configuration of the vortex street. From this it follows that an asymmetric vortex street is stable, provided the parameters κ and λ of the configuration satisfy the relation (*). For $\lambda = 1/2$, the well-known necessary condition of stability for von Kármán's alternating vortex street is obtained. Symmetric vortex streets turn out to be unstable. It should be noted, however, that this theory of stability of generalized (two parameter) vortex streets for small displacements is only an approximative one, and a revision of this theory will be given in a future paper in the same Journal.

E. Leimanis, Canada

4200. Wehner, H., Considerations about flow conditions of a traveling Kármán vortex street (in German), *Z. Meteor.* **4**, 7/8, 248–250, July/Aug. 1950.

Author evaluates the velocity field at a fixed point inside and far away from a traveling von Kármán vortex street. It is shown that inside, near the center line, the velocity component parallel to the street (u) remains practically constant, but the transverse component (v) varies harmonically. Outside and far away from the street, both components vary harmonically and the resulting velocity vector rotates steadily. It is implied that these results have a bearing on observed atmospheric velocity fluctuations. A. D. Young, England

4201. Kaufman, W., Time history of the coiling phenomenon of an unstable surface of discontinuity of finite width (in German), *Ing.-Arch.* **19**, 1, 1–11, 1951.

The starting point of this analysis is Helmholtz' law about the vortex motion in an ideal fluid. The basic assumptions are: (a) The velocity field near the partially coiled vortex is that of a circular vortex kernel. (b) When the coiling phenomenon has started a very short time before, the vortex sheet can be considered to have an infinite width.

Results are compared with measurements of the vortex configuration behind a rectangular wing. H. G. Loos, Holland

4202. Ghosh, N. L., Equilibrium of rotating fluid-bodies in confocal stratifications. I, II, *Bull. Calcutta math. Soc.* **42**, 4, 227–248, Dec. 1950.

In 1950, Dive ["Rotations Internes des Astres Fluides," Paris, 1950] treated the problem of rotating equilibrium of fluid bodies in confocal stratifications. He obtained the following results: (1) Whenever the stratifications are in spheroids confocal with the boundary, equilibrium under self-gravitation is possible for any law of variation of the density, provided the density diminishes outward continuously. (2) In every such equilibrium the angular velocity is constant over the same stratification and, hence, the shells of equal density turn as a whole. (3) The angular velocity varies from shell to shell and continuously diminishes outward under the same provisions as in (1).

In part I of present paper, author gives a complete integration of the same problem and obtains the general expressions for the potential, the angular velocity, and the pressure. In addition to obtaining the results arrived at by Dive, the analysis also serves to bring out certain characteristic features of the problem. In part II author assumes a certain law for the density variation and carries out the integrations in closed form in terms of ordinary functions.

Regarding method of approach, author first calculates the gravitation potential for any law of density variation, then uses the results obtained in Euler's hydrodynamic equations of motion with the angular velocity and the pressure as unknowns, the fluid being assumed an ideal fluid, and the radial velocity from the rotation axis as well as the velocity along that axis being as-

sumed zero. The result for the angular velocity shows that:

1. The solution does not satisfy the Navier-Stokes equation for v_θ (where θ is one of the cylindrical coordinates) since the term involving viscosity is not zero unless the viscosity is zero. Therefore, author's specification of ideal fluid is necessary.

2. There is vorticity in the fluid which is assumed to be ideal. This means the motion cannot have started from rest or from any irrotational state, in which cases the irrotationality would persist. The genesis of the equilibrium state therefore remains an open question. Chia-Shun Yih, USA

4203. Spraglin, W. E., Flow through cascades in tandem, *Nat. adv. Comm. Aero. tech. Note* 2393, 44 pp., June 1951.

A formal solution is presented for the incompressible, inviscid two-dimensional flow around two unstaggered cascades in tandem. Functions are given for conformally mapping the tandem cascade onto the region between two concentric circles, in which region there are suitably placed flow singularities.

Gerald Nitzberg, USA

4204. Dresden, D., On the theory of the stuffing box in reciprocating machines (in Dutch), *Ingenieur* **62**, 32, W.69–W.71, Aug. 1950.

For a given periodic total pressure drop, pressure and mass rate of flow of the liquid are calculated as functions of time and of place in the box, assuming laminar flow and neglecting the motion of the piston-rod and liquid-inertia effects. Liquid sorption by the packing material is assumed to vary proportionally to the pressure. Because of these simplifications, only a first approximation is arrived at. By integrating the liquid flow over half a period, it is found that liquid leakage is small for high frequencies and for materials with a high liquid sorption capacity. Siegers [*Ingenieur* **62**, 45, W.104–W.105, Nov. 10, 1950], who used the same simplifications, showed that the latter conclusion is due to a mathematical error and that total leakage is independent of sorption capacity. H. A. Vreedenberg, Holland

4205. Broer, L. J. F., The movement of a viscous fluid in a gap with porous walls (in Dutch), *Ingenieur* **63**, 24, W.61–W.65, June 1951.

The problem of the stuffing box in reciprocating machines is treated more rigorously than by Dresden [see preceding review]. The exact solution of the equations for laminar flow, including the liquid-inertia term, is discussed. Transverse motion of the liquid and the nonlinear convective terms are neglected; calculation of some dimensionless groups shows when this is permissible. Liquid resistance in the packing material is represented by a term in the differential equation for the amount of liquid absorbed. With increasing resistance the damping of the periodical pressure and velocity fluctuations by the stuffing box increases at first, but after passing a maximum it decreases rapidly. Damping decreases as the result of liquid inertia, whereas the motion of the shaft has no influence.

H. A. Vreedenberg, Holland

4206. Francis, J. R. D., The aerodynamic drag of a free water surface, *Proc. roy. Soc. Lond. Ser. A*, **206**, 1086, 387–406, May 1951.

Water waves were created from wind in model tunnel (7.5 cm wide, 7 m long) seen at sea, (a visual observation). Shear-stress coefficient is found to increase nearly linearly with wind speed and was obtained by measuring slope of the mean water surface. Drag increases nearly with the cube of wind speed. Experimental data are given for shear stress as a function of wind speed. Author gives laboratory method of increasing effective fetch by adding circular air paddle entrance to straight wind tunnel with exhaust fan. Mean wave height and mean wave period generated with fan only agree with available results (Reviewers note: see "Limited fetch studies" by J. W. Johnson, E. K. Rice, and C. L. Bretschneider, unpublished). Wind-waves speed at given wave length is greater than given by Kelvin's equation. Evidence of hitherto unreported type of air-flow distribution over water waves is presented. This phenomenon may account for some variations in field and laboratory results. Author points out limitation of his experiments as well as areas of satisfactory agreement with others. Jack R. Morison, USA

4207. Ingebo, R. D., Vaporization rates and heat-transfer coefficients for pure liquid drops, *Nat. adv. Comm. Aero. tech. note* 2368, 32 pp., July 1951.

Nine liquids, having latent heats from 50 to 500 g cal/g, were evaporated in an air stream at 30 to 500 C. The liquid was fed by means of a hypo needle to the center of a cork sphere, 0.688 cm in diameter. Correlations were made by equations obtained from dimensional analysis in which (Re Sc)$^{0.6}$ appears. Liquid feed temperature had a negligible effect on the experimental heat-transfer coefficient. The applicability of the equations was extended by approximating Δt from boiling-point data instead of relying on the availability of wet-bulb temperature data.

Correlation of experimental data was good; points were spread over a range of 20% of the predicted values.

H. E. Robison, USA

4208. van Heemert, A., The calculation of downwash fields for a lifting plane in steady flow, *Nat. LuchtLab. Amsterdam Rap.* F. 51, 42 pp., 1949, published 1951.

Author evaluates downwash integral of incompressible-flow lifting-surface theory for doublet distributions over strips of the surface with sides parallel to the free stream and with intensities which vary linearly in the spanwise direction and are represented in the chordwise direction by the terms of the series expansion used in two-dimensional thin-airfoil theory. He proposes to calculate tables of these integrals and to use them in conjunction with simple numerical-integration techniques for the solution of the direct and inverse problems of lifting-surface theory.

F. W. Diederich, USA

4209. van Heemert, A., A generalization of Prandtl's equation, *Nat. LuchtLab. Amsterdam Rap.* F.76, 47 pp., Jan. 1951.

Author derives equations, which reduce to Prandtl's lifting-line equation in case of unswept wings, for vorticity distribution of yawed and swept wings. He attempts to surmount difficulties experienced by previous investigators [e.g., Theilheimer, *J. aero. Sci.* **10**, 3, 101–104, Mar. 1943; Lin, *J. aero. Sci.* **11**, 3, 195–196, July 1944] with the singularities in downwash integral by retaining chordwise variation of vorticity and imposing restrictions on the spanwise variation near the root of a swept wing. Effects of approximations made are difficult to assess; results obtainable by means of generalized equations are stated to be satisfactory. F. W. Diederich, USA

4210. van Heemert, A., Application of the generalized Prandtl equation to an elliptic plane wing under yaw, *Nat. LuchtLab. Amsterdam Rap.* F.77, 15 pp., Mar. 1951.

In a previous paper [see preceding review] author derived generalized equation applicable to swept and yawed wing, which in the case of unswept wings reduces to Prandtl's lifting-line equation. In this paper the equation is solved for a yawed elliptic wing and the results compared to Krienes' [*Z. angew. Math.*

Mech. **20**, 2, 65–88, April 1940, translated as *Nat. Adv. Comm. Aero. tech. Memo.* 971, Mar. 1941]. The lift and pitching moment are in good agreement, the difference being of the order to be expected in the results of a lifting-surface and a lifting-line solution, respectively, but the small rolling moment apparently cannot be calculated by the method as given in this paper.

F. W. Diederich, USA

4211. Bellman, R., On an equation occurring in the harmonic analysis of viscous fluid flow, *Quart. appl. Math.* **9**, 2, 218–223, July 1951.

In a previous paper [AMR **1**, Rev. 859] by J. Kampé de Fériet, an integrodifferential equation was found for the Fourier transform of the vorticity in two-dimensional incompressible viscous flow. In present paper, author proves that the system consisting of this equation and an initial condition possesses a solution if the initial value of the unknown is sufficiently small. As a by-product of the proof, author also finds an upper bound for the solution. Method of successive approximations is used in the proof of the existence theorem. As usual with this approach, the establishment of existence of the solution in the mean time also actually yields the solution. Proof of uniqueness is indicated briefly.

Chia-Shun Yih, USA

4212. Libby, P. A., and Reiss, H. R., The design of two-dimensional contraction sections, *Quart. appl. Math.* **9**, 1, 95–98, Apr. 1951.

The design of two-dimensional incompressible contraction shapes is considered in the case where the velocity increases monotonically along the boundary from the low to the high-speed end. The method starts from a prescribed velocity distribution satisfying the above monotonic condition along the boundary streamlines in the hodograph plane. The coordinates in the physical plane are found by integration.

G. M. Lilley, England

4213. Golubeva, O. V., The equations of two-dimensional motion of an ideal fluid on a curvilinear surface and their application in the theory of filtration (in Russian), *Prikl. Mat. Mekh.* **14**, 287–294, 1950.

Author writes (in Gaussian coordinates) the equations of a two-dimensional steady motion of a perfect fluid for the case when the fluid moves along a given surface. Similarly she rewrites the equations of filtration. Under suitable assumption she ends up with Beltrami's equations and obtains some particular solutions (said to be of importance for filtration theory) for the case when the surface is one of revolution, so that the conformal mapping on a plane can be accomplished by quadratures. L. Bers, USA

4214. Millsaps, K., and Pohlhausen, J., Heat transfer by laminar flow from a rotating plate, *J. aero. Sci.* **18**, 5, 354–355, May 1951.

Note in Readers' Forum; a more detailed presentation is to be given in a later paper. Ed.

4215. Fogarty, L. E., The laminar boundary layer on a rotating blade, *J. aero. Sci.* **18**, 4, 247–252, Apr. 1951.

By writing the Navier-Stokes equations for steady motion referred to the rotating orthogonal coordinate system formed by the blade leading edge and the axis of rotation, and by introducing the usual boundary-layer assumptions, author develops equations of motion for the incompressible boundary layer at large distances from the axis of rotation. Equations are found to be similar to those found by Prandtl, Jones, and Sears for case of infinite yawed wing. Author calculates two cases (flat plate and symmetrical cylinder) and finds that effects of rotation on unseparated laminar boundary layer are small.

Alvin H. Sacks, USA

4216. [illegible], V. A., and Dityakin, Y. F., Unstable capillary waves on surface of [illegible] of two viscous fluids, *Nat. adv. Comm. Aero. tech. Mem.* [illegible], [illegible] pp., Apr. 1951

Author attacks problem of disintegration of jet into drops of many sizes. Indication of breakup mechanism is found by applying known stability-investigation method [AMR **3**, Rev. 1118]. Two-dimensional plane-disturbance differential equation is integrated directly. Eigenvalues found for oversimplified case give indication that a spectrum of drop sizes should accompany disintegration. Reviewer believes paper is pertinent to research in stability of interfaces having application to multiphase flow in tubes and in porous media, to formation of ocean waves, and to behavior of jets. Alan D. K. Laird, USA

Compressible Flow, Gas Dynamics

(See also Revs. 4046, 4051, 4241, 4242, 4243, 4256, 4261, 4266, 4270 4278, 4280, 4286, 4292, 4311)

4217. Sauer, R., Introduction to theoretical gas dynamics, 2d ed. (Einführung in die theoretische gasdynamik), Berlin, Springer-Verlag, 1951, viii + 174 pp., 107 figs. DM 16.50.

In this second edition of the author's treatise on gas dynamics, first published in 1943, results of recent research have been included, and chapters have been added on the application of the method of characteristics to rotational flow and to three-dimensional flow with axial symmetry, and on three-dimensional supersonic flow around bodies of revolution and around airfoils of finite width. The concise presentation of the subject will continue to be a valuable aid to the engineer engaged in work on supersonic flow. E. F. Lype, USA

4218. Lighthill, M. J., Reflection at a laminar boundary layer of a weak steady disturbance to a supersonic stream, neglecting viscosity and heat conduction, *Quart. J. Mech. appl. Math.* **3**, part 3, 303–325, Sept. 1950.

Boundary layer is replaced by nonviscous shear flow in which Mach number $M(y)$ varies continuously and monotonically from zero at the wall, $y = 0$, to the free-stream value M_1 at $y = \delta$. In contrast with Robinson's model [AMR **4**, Rev. 1662] the present linearization allows for disturbance vorticity and leads to a second-order equation in x and y for the pressure coefficient ω (with an extra first-order term inside the layer $y < 0$). Linearization and the condition $M(0) = 0$ cause the pressure to remain constant along the wall, $\omega = 0$ for $y = 0$, a "boundary condition which may be the cause of the disagreement with experiment." Application of Fourier integral technique yields an ordinary differential equation with a regular singular point at $y = 0$. Asymptotic behavior of solutions is established in the transform plane with special care in regions near $M(y) = 1$ and is interpreted in the physical plane. A pressure (or pressure gradient) jump in the incident wave propagates along characteristics in the supersonic part of the layer, increasing in strength until it reflects from the sonic line as a logarithmically infinite jump of the same sign. This linearized reflection is interpreted for a real fluid as a sharp pressure increase followed by a slower decrease, i.e., compression-expansion wave which, author believes, can be identified in experimental reflections of weak shocks from *turbulent* boundary layers. The theoretical upstream influence of the incoming wave is shown to be negligibly small, irrespective of the Mach number profile, $M(y)$. This disagrees with experi-

mental results for laminar layers and with theoretical results of Tsien and Finston [AMR **3**, Rev. 2711] based on a cruder flow model (but agrees with Robinson's theory). The contradiction with Tsien-Finston is analyzed. In discussing the general shock boundary-layer interaction, four flow models of increased complexity are advanced. Analysis of a model which accounts for viscosity effects in a sublayer at the wall is to appear shortly, obviating the criticism of the weaknesses of the present model (boundary condition $\omega = 0$; perturbation procedure on "slow" flow at wall). M. V. Morkovin, USA

4219. Chang, C.-C., Applications of von Kármán's integral method in supersonic wing theory, *Nat. adv. Comm. Aero. tech. Note* 2317, 71 pp., Mar. 1951.

Author presents details, physical interpretation, and further applications of the von Kármán-Fourier integral method [AMR 1, Rev. 133], utilizing the acoustic analogy where the time sequence of acoustic oscillators corresponds to harmonic line sources or doublets for supersonic finite-wing theory.

General solution for wave drag is given for arbitrary planforms of swept, tapered wings with symmetrical double wedge profiles. Also for these wings, downwash and angle-of-attack distribution is found for a constant lift distribution. Results have been given previously by others, but are now unified by single integral approach that is also applicable to downwash distribution for any span or chordwise lift distribution. E. V. Laitone, USA

4220. Cabannes, H., The problem of the detached shock wave for rotational flow (in French), *C. R. Acad. Sci. Paris* **232**, 8, 686–687, Feb. 1951.

If a detached bow wave in axially symmetric flow is approximated by a fourth-order equation, a method is described for finding the corresponding body shape. Depending on the equation chosen, either an annular or convex body results. Reasonable agreement is found with experimental data on round-nosed objects, especially in view of the experimental observation that the bow-wave shape is quite insensitive to the body shape.

Wayland Griffith, USA

4221. Burgers, J. M., Aerodynamical description of elementary expansion phenomena and shock waves, *Intern. Union theor. appl. Mech. and intern. astron. Union*, Problems of cosmical aerodynamics, 59–70, 1951.

4222. Burgers, J. M., The properties of a shock wave in a gas with decreasing density, *Intern. Union theor. appl. Mech. and intern. astron. Union*, Problems of cosmical aerodynamics, 98–106, 1951.

4223. Kuo, Y.-H., Two-dimensional transonic flow past airfoils, *Nat. adv. Comm. Aero. tech. Note* 2356, 48 pp., May 1951.

The solution of the problem of compressible flow past a thin Joukowsky airfoil, with and without circulation, is carried out mathematically using hodograph theory and conformal mapping. The mapped flow is that of an approximate incompressible flow past the airfoil, derived from a source and sink. No numerical results are given. Lester L. Cronvich, USA

4224. Mitchell, A. R., and Rutherford, D. E., Application of relaxation methods to compressible flow past a double wedge, *Proc. roy. Soc. Edinburgh, Sec. A*, **63**, part II, no. 11, 139–154, 1951.

Paper shows that relaxation method can be used to solve certain transonic-flow problems; in particular, the problem of a uniform low subsonic velocity stream impinging on a two-dimensional symmetrical double wedge. An asymmetrical supersonic region was isolated near the wedge apex, and the streamlines and Mach number values within this supersonic region were determined by application of the relaxation technique. A breakdown of this technique is to be expected in the neighborhood of the sonic line, but in the above problem this difficulty was surmounted. Simon Ostrach, USA

4225. Germain, P., Application of homographic approximation to the study of transonic flow (in French), *C. R. Acad. Sci. Paris* **232**, 20, 1811–1813, May 1951.

Author takes hodograph equation of compressible flow in normalized form $k(\sigma)\psi_{\theta\theta} + \psi_{\sigma\sigma} = 0$ and replaces $k(\sigma)$ by the approximation $\sigma/\sigma + 1$. Resulting equation, being of Tricomic type, has similar solutions, two of which are given in closed form. Application to transonic flow is suggested but not presented. Robert E. Street, USA

4226. Thomas, T. Y., Supersonic airfoils, *Dept. Navy, Monthly Res. Rep. Off. Nav. Res.*, 2 pp., Apr. 1951.

In his previous review of author's paper on the determination of pressure on curved bodies behind shocks [AMR **4**, Rev. 1661], the reviewer mentioned that author's method may fail when separation process begins. In the present note, author attempts to meet this situation by a brilliant assumption that the Mach angles tend to remain invariant during the separation process. A few graphs show an excellent agreement between the calculated and experimental pressure curves. No calculations are discussed. M. Z. Krzywoblocki, USA

4227. Sears, W. R., and Tan, H. S., The aerodynamics of supersonic biplanes, *Quart. appl. Math.* **9**, 1, 67–76, Apr. 1951.

Practical importance of supersonic biplane airfoils lies in possibilities of reducing wave drag by using thinner wing sections than is possible in monoplane arrangement, and in providing for proper wave interactions between upper and lower wing (i.e., Busemann two-dimensional scheme of zero drag at zero lift). Paper is a brief report on investigation (thesis of second author) of supersonic biplanes of finite span having same rectangular wings. Linear theory is used and Busemann relationship between gap, chord, and Mach number is assumed. Study proceeds on Evvard's "source and sink"-diaphragm method [AMR **1**, Rev. 689]. Authors derive integral formulas for velocity potential. Results are applied to finite Busemann biplane at zero lift and $M = 2^{1/2}$, the wing sections consisting, according to linear treatment, of two isosceles triangles pointing at each other. Spanwise wave-drag distribution near wing tip is calculated. Paper is intended as a brief account giving a line of investigation still in progress, rather than as a memorandum for convenient study.

Pierre Schwaar, Switzerland

4228. Truesdell, C. A. T., On Ertel's vorticity theorem *Z. angew. Math. Phys.* **2**, 2, 109–114, Mar. 1951.

In this short note the kinematical behavior of Ertel's vorticity theorem is explored somewhat. The theorem is also extended to viscous compressible fluids. Two special cases are given.

Chieh-Chien Chang, USA

4229. Young, A. D., The equations of motion and energy and the velocity profile of a turbulent boundary layer in a compressible fluid, *Coll. Aero. Cranfield Rep.* **42**, 14 pp., Jan. 1951.

After setting up the equations of motion, continuity, and energy for a turbulent compressible fluid in tensor notation, author considers special case of two-dimensional boundary layer. He makes the usual boundary-layer assumptions and reduces the

problem step by step to the turbulent boundary layer on infinite plate in steady parallel flow, with zero external pressure gradient. By using the mixing length theory, he finally obtains an expression for the boundary-layer profile which is analogous to the log law of incompressible flow. Two further assumptions are made for the evaluation: (1) The constants linking the mixing length and the distance from the wall are equal in incompressible and compressible flow. (2) After results by Monoghan, the ratio of the frictional velocities in incompressible and compressible flow is equal to the $^1/$ power of the ratio of the wall temperature T_w and the free-stream temperature T_1. Thus a relation for the velocity profiles is found which is evaluated for the Mach numbers 2.5 and 5.0 and T_w/T_1 of 0.25; 1.0 and 10. Author concludes that "for Mach numbers of the order of 2.5 or less and for a wide range of heat-transfer conditions the velocity profile in the turbulent boundary layer will differ very little from that for an incompressible fluid and the same Reynolds number. For higher Mach numbers, however, small differences will become apparent, particularly for case of considerable heat transfer from the surface to the fluid."

Heinrich Ramm, USA

4230. Tucker, M., Approximate calculation of turbulent boundary-layer development in compressible flow, *Nat. adv. Comm. Aero. tech. Note* 2337, 42 pp., Apr. 1951.

Paper uses the conventional approaches to the turbulent boundary-layer prediction problem. Fluid properties are evaluated at a temperature halfway between that at the wall and that in the free stream. Tables are given for Mach numbers between 0.1 and 10, which reduce boundary-layer calculations to a routine procedure. These will be of value in engineering work where an estimate of boundary-layer thickness is desired. The method is not expected to be valid for flows approaching separation.

In the field covered by this paper there is no lack in variety of hypotheses concerning turbulent boundary layers, but rather an absence of basic data to check the assumptions that have already been made. Author and reviewer appear not to be in disagreement on this point.

Francis H. Clauser, USA

4231. Neumann, E. P., and Lustwerk, F., High-efficiency supersonic diffusers, *J. aero. Sci.* **18**, 6, 369–374, June 1951.

Paper deals with experimental results obtained at the Gas Turbine Laboratory, Massachusetts Institute of Technology, concerning performance of diffusers of variable geometry designed for wind-tunnel applications.

A small-scale diffuser was tested, having adjustable side walls in order to permit reduction of the throat area after starting. The percentage of stagnation pressure recovered was 79% at $M = 2.22$ and 53% at $M = 2.92$, without boundary-layer suction. Diffuser efficiency, defined as the ratio of isentropic increase in enthalpy (computed between the temperature at the diffuser entrance and the one corresponding to the effective exit pressure on the basis of an ideal isentropic process) to the change in kinetic energy in the diffuser, was 0.87 at $M = 2.22$ and 0.72 at $M = 2.92$. Scale effects are examined: experimental data bring authors to believe that the obtained results can be well extrapolated to larger scale application.

A type of multicellular diffuser was also tested in order to decrease the required length of the shock region: experiments on fixed-geometry diffusers indicate the possibility of reducing by 25% the over-all length without important losses in stagnation-pressure recovery.

To conclude, results are quite in agreement with the theory. They confirm the possibility of achieving large reductions in losses produced by strong shocks; consequently, significant savings in power required to operate a wind tunnel can be obtained over conventional designs.

Angelo Miele, Argentina

4232. Ferri, A., and Nucci, L. M., Preliminary investigation of a new type of supersonic inlet, *Nat. adv. Comm. Aero. tech. Note* 2286, 41 pp., Apr. 1951.

Paper deals with the annular type of supersonic air inlet having a central conical-nosed body, with all supersonic deceleration occurring outside the inlet. Behavior of some possible arrangements is analyzed numerically using theory of flow past cones and method of characteristics. Wind-tunnel measurements confirm theoretical results and show that pressure-recovery ratios of 0.92 at $M = 1.7$ and 0.86 at $M = 2.1$ can be obtained. With fixed geometry of inlet, high pressure recoveries are possible for fairly large ranges of mass flow and flight Mach number, making the inlet suitable for use on supersonic airplanes. External drag is larger than for some other types of inlet.

Paper was first issued as classified document in 1946.

W. A. Mair, England

4233. Naylor, V. D., The critical flow of a gas through a convergent nozzle, *Aircr. Engng.* **23**, 268, 160–162, June 1951.

The classical equations of gas flow through a converging-diverging nozzle have been extended to include the effect of friction. The basic assumptions are that the small stage efficiency η_∞ of expansion (and/or compression) is constant throughout the total expansion (and/or compression) and that the local acoustic velocity is dependent on η_∞. The constancy of η_∞ leads to a polytropic law pv^n = constant, with the condition that n is the ratio of specific heats γ when $\eta_\infty = 1$. The equations of motion, continuity, conservation of energy, and constant mass are written to include the effects of friction, expressed in terms of η_∞. From thence the critical flow conditions are determined. At $\eta_\infty = 1$ these reduce to usual isentropic channel flow equations. Graphs are given of the mass flow, pressure, velocity, and area relationships in the nozzle for values of $\eta_\infty = 0.8, 0.9$, and 1. The exit velocity which is expressed in terms of the "critical" or throat velocity would perhaps be more useful if expressed in terms of the acoustic velocity at the exit. This calculation could easily be made and would enable a more ready comparison between the Mach number of frictionless isentropic flow and the "effective Mach number" for flow whose efficiency is less than 1.

There are a few typographical errors in the article.

Ione D. V. Faro, USA

4234. Mirels, H., A lift-cancellation technique in linearized supersonic wing theory, *Nat. adv. Comm. Aero. Rep.* 1004, 11 pp., 1951.

See AMR **4**, Rev. 2122.

4235. Struminsky, V. V., Sideslip in a viscous compressible gas (Translation from Russian), *Nat. adv. Comm. Aero. tech. Memo.* 1276, 8 pp., Apr. 1951.

By an analysis of the Navier-Stokes equation, it is shown that the aerodynamic coefficients of an infinite rectangular swept wing in a compressible flow can be determined from the aerodynamic coefficients of the unswept wing. The compressible flow is considered to be either isothermal or adiabatic. In the case of general flow, a three-dimensional boundary-layer theory is developed and applied to the special case of a swept flat plate.

C. T. Wang, USA

4236. Stocker, P. M., Supersonic flow past bodies of revolution with thin wings of small aspect ratio, *Aero. Quart.* **3**, part 1, 61–79, May 1951.

Methods and results of general slender-body theory are used to calculate effect of wing-body interference for pointed body of

revolution with (1) thin, small aspect-ratio wings of symmetrical section at zero incidence to body and zero angle of attack, (2) plane, small aspect-ratio wings at incidence to body, (3) plane, small aspect-ratio fin at incidence to body. Pressure distribution over wings for special case of (1) is exhibited. Formulas are given for over-all aerodynamic forces in (2) and (3). In case (2) it is shown that, for given lift, drag is a minimum when wings are at zero incidence to body. G. N. Ward, England

4237. Reissner, E., On the theory of oscillating airfoils of finite span in subsonic compressible flow, *Nat. adv. Comm. Aero. Rep.* 1002, 9 pp., 1950.

See AMR **3**, Rev. 2724.

4238. Soule, H. V., and Sabol, A. P., Development and preliminary investigation of a method of obtaining hypersonic aerodynamic data by firing models through highly cooled gases, *Nat. adv. Comm. Aero. tech. Note* 2120, 38 pp., July 1950.

Experiments are described of firing conical models through a gas at rest and obtaining a Mach number of 6.7 and Reynolds numbers as high as 5 million. Apparatus consisted mainly of (1) a test chamber containing cooled nitrogen gas where the sonic velocity was low, and (2) a model, fired from a 0.22-caliber gun, which passed through the test chamber at a nominal velocity of 4200 fps. Suitable schlieren and shadowgraph optical systems were used to photograph the model in flight, and a chronograph activated by photocells measured the bullet-model velocity.

Results include comparison of shock angles obtained experimentally from 45° to 60° cones with theoretical values obtained where a constant value of γ, the ratio of the specific heats of the gas, was assumed to hold. An over-all accuracy of $\pm 4.5\%$ is given for Mach number determinations with this equipment under the conditions of the experiment reported.

F. K. Hill, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 4218, 4229, 4230, 4271)

4239. Schlichting, H., Amplitude distribution and energy balance of small disturbances in plate flow, *Nat. adv. Comm. Aero. tech. Memo* 1265, 44 pp., Apr. 1950.

Translation from *Nachrichten Gesell. Wissen. Göttingen* **1**, 4, 1935.

4240. Burgers, J. M., Correlation problems in a one-dimensional model of turbulence. I, *Proc. kon. Ned. Akad. Wet.* **53**, 3, 247–260, 1950.

Author studies the equation

$$\partial v/\partial t + v\partial v/\partial y = \nu\partial^2 v/\partial y^2, \qquad [1]$$

in order to gain insight into the problem of turbulence. Developments, along the lines of the correlation theory, analogous to the usual ones in the three-dimensional case are carried out. To obtain additional information, certain exact particular solutions of [1] are obtained and investigated. These solutions demonstrate clearly the increase in the gradient of velocity, the limitation of this process by the effect of viscosity, and the coalescence of the sharp fronts thus formed. Author then proceeds to study correlation functions corresponding to these solutions. [Reviewer's remark. The theory developed in this paper is, of course, not intended to deal with another aspect of the phenomenon of turbulence, namely, the stretching of vortex filaments. On the other hand, the results obtained here may also be used to demonstrate several aspects of the phenomenon of shock formation.] C. C. Lin, USA

4241. Tifford, A. N., Simplified compressible laminar boundary-layer theory, *J. aero. Sci.* **18**, 5, 358–359, May 1951.

Intent of note is to emphasize that, in many current applications, the characteristics of the laminar boundary layer at any point on a surface may be determined within engineering precision on the basis of the simpler analyses extant in the literature, assuming fluid properties throughout the boundary layer to be the same as at that particular point.

From author's summary

4242. Timman, R., Method of characteristics and the calculation of the laminar boundary layer in a three-dimensional flow (in French), *Nat. LuchtLab. Amsterdam Rap.* F.62, 7 pp., Apr. 1950.

The von Kármán-Pohlhausen method is generalized to arbitrary three-dimensional shapes. Transforming incompressible boundary-layer equations from curvilinear coordinates to natural coordinates of local external flow, and defining displacement-thickness vector and momentum-thickness tensor, lead to two relations which are a generalization of von Kármán integral relation. Assuming Pohlhausen's form for streamwise velocity profile, and analogous form for cross-stream profile, yields pair of first-order partial differential equations. Author suggests numerical solution by method of characteristics; necessary initial values at stagnation points will be considered in subsequent paper.

Reviewer believes author's basic boundary-layer equations apply only to cylinders; Howarth [AMR **4**, Rev. 3640] has shown that curvature must appear in general equations. Significance of variable ρ throughout analysis is not clear to reviewer.

Milton D. Van Dyke, USA

4243. Kuerti, G., The laminar boundary layer in compressible flow, *Advances appl. Mech.* **II**, 21–92, 1951. [Academic Press, Inc., N. Y.]

Paper is a guide to literature in this field. Only plane problem is considered. Author discusses boundary-layer equations and various transformations which render analytical approach simpler under certain assumptions (e.g., Prandtl number unity), which adapt equations to use of approximate methods (e.g., Pohlhausen method), or which prepare equations for a computer. Results of various authors are discussed. Some thirty references are given, the latest being dated 1949.

John A. Lewis, USA

4244. Lee, T. D., Difference between turbulence in a two-dimensional fluid and in a three-dimensional fluid, *J. appl. Phys.* **22**, 4, p. 524, Apr. 1951.

By using the vorticity equation for a two-dimensional motion, author shows that the assumption of an inertial subrange in the equilibrium spectrum of turbulence leads to a contradiction not occurring when three-dimensional turbulence is considered. Reviewer agrees with the conclusion that the spectrum law $F(k)$ is incompatible with two-dimensional motion, but believes that this can be shown better by considering the nature of the energy-transfer function. A. A. Townsend, England

4245. Freeman, J. C., Jr., Note on the minimum critical Reynolds number and form parameter, *J. aero. Sci.* **18**, 5, 350–351, May 1951.

In the note it is shown that, if sufficient suction of the type $v_0(x) \sim x^{(m-1)/2}$ is introduced on a flat plate with pressure

gradient (such that the free-stream velocity $u_0(x) = U_0x^m$), R_1 (minimum critical Reynolds number based on the displacement thickness) for many resulting profiles is essentially determined by H (form parameter $= \delta_1/\vartheta$, where δ_1 is displacement thickness, ϑ momentum thickness). From author's summary

4246. Limber, D. N., Numerical results for pressure-velocity correlations in homogeneous isotropic turbulence, *Proc. nat. Acad. Sci. Wash.* **37, 4,** 230–233, Apr. 1951.

Author examines the mean value $1/\rho\, \overline{p'u_iu_j}$ by the methods used by the reviewer [AMR 4, Rev. 3947] to examine $1/\rho\, \overline{p'u_i}$, where u_i is a turbulent velocity component at one point and p' is the simultaneous pressure at another point distance r away. The basic assumption is that fourth-order velocity-product mean values are related to second-order mean values as for a jointly normal distribution of the velocities at two points. Numerical results are given for the case of infinite Reynolds number, for which the velocity correlation is known empirically. An interesting feature is that $1/\rho\, \overline{p'u^2_\alpha}$ where α denotes a component parallel to the line joining the two points, is negative for small values of r (presumably reflecting the kind of anti-correlation between p and u^2_α which occurs in Bernoulli's theorem) and becomes positive at larger values of r. G. K. Batchelor, England

4247. Monaghan, R. J., Comparison between experimental measurements and a suggested formula for the variation of turbulent skin-friction in compressible flow, *Aero. Res. Counc. Lond. curr. Pap.* 45, 16 pp., 6 figs., Feb. 1950, published 1951.

Measurements made in the turbulent boundary layer on a flat plate, with zero heat transfer, at a Mach number of 2.45, led author to believe that the skin-friction coefficient under such conditions could be related to the skin-friction coefficient in incompressible flow by the following equations:

$$C_{F_i} = C_{F_w}$$

$$\text{when} \qquad \mathrm{Re}_i = \mathrm{Re}_w(T_1/T_w) \qquad [1]$$

The subscripts i and w are here used to denote the values in the incompressible case and measured at the wall in the compressible case, respectively, for the skin-friction coefficient, the Reynolds number, and the temperature. Where no subscript is used the value is that of the free stream.

Results of tests made by Wilson, Young and Thompson at the Defense Research Laboratory of the University of Texas are used to provide a check on Eq. [1] for Mach numbers from 1.73 to 2.25 and for Reynolds numbers from 4 to 20 million. Measurements were made on a flat plate under conditions of zero heat transfer. By employing the von Kármán value for C_{F_i}, i.e., $C_{F_i} = 0.074\ (\mathrm{Re}_i)^{-1/5}$, good agreement is found between the experimental data and the values computed by Eq. [1].

Test results of Humble, Lowdermilk, and Grele of the NACA Lewis Flight Propulsion Laboratory at Cleveland, Ohio, show good agreement with those obtained from the empirical formula for mean skin-friction coefficient in the case of pipe flow which was given by Blasius and modified by means of the method of Eq. [1]. Furthermore, the NACA test results show that similar equations may be used successfully to derive the heat-transfer coefficient for pipe flow. The data used in the checks were in the range of Reynolds numbers from 7,000 to 50,000 and for temperature differences up to 684 C. Ione D. V. Faro, USA

4248. Batchelor, G. K., Note on a class of solutions of the Navier-Stokes equations representing steady rotationally-symmetric flow, *Quart. J. Mech. appl. Math.* **4,** part I, 29–41, Mar. 1951.

Author discusses the steady flow of an incompressible viscous fluid in regions bounded by one or two concentric infinite rotating disks. Then, the axial velocity depends on the axial coordinate z alone and the Navier-Stokes equations reduce to two nonlinear total differential equations in z for the axial and angular velocities, with appropriate boundary conditions along the disks and/or at infinity. Without solving these equations, author describes qualitatively the flow pattern for single disk problems with disk angular velocity smaller, larger, and of opposite sign from that at infinity, and double disk problems with and without angular velocity sign reversal. When Reynolds number based on disk separation becomes large, the flow becomes axial with boundary-layer-like regions near disks, and a sort of inner friction layer in case of velocity reversal. Leon Trilling, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 4208, 4219, 4235, 4275)

4249. Lichtenstein, J. H., Effect of horizontal-tail size and tail length on low-speed static longitudinal stability and damping in pitch of a model having 45° sweptback wing and tail surfaces, *Nat. adv. Comm. Aero. tech. Note* 2382, 32 pp., June 1951.

An investigation has been conducted in the Langley stability tunnel to determine the effects of horizontal tails of various sizes and at various tail lengths when located at the center line of the fuselage on low-speed static longitudinal stability, and steady-state rotary damping-in-pitch of a complete model with wing and tail surfaces having the quarter-chord lines swept back 45° and aspect ratios of 4.

Results of the investigation show that, in agreement with analytical considerations, contribution of the horizontal tail to static longitudinal stability is directly related to tail size and length; whereas the contribution to rotary damping-in-pitch is directly related to tail size and square of the tail length.

For low angles of attack, the wing downwash reduced the effectiveness of the tail in contributing static longitudinal stability of approximately one half; whereas the wing had practically no effect on the effectiveness of the tail in contributing to the rotary damping-in-pitch. For the tail positions investigated herein, static longitudinal stability was slightly greater near the stall than at an angle of attack of 0°; whereas damping-in-pitch was somewhat less near the stall than at an angle of attack of 0°.

At an angle of attack of about 10°, the static longitudinal stability of the wing-fuselage combinations changed adversely. The magnitude of this change was slightly increased by the addition of tail area at the shortest tail length, but was decreased by addition of tail area at the longest tail length.

From author's summary

4250. Lichtenstein, J. H., Effect of horizontal-tail location on low-speed static longitudinal stability and damping in pitch of a model having 45° sweptback wing and tail surfaces, *Nat. adv. Comm. Aero. tech. Note* 2381, 26 pp., June 1951.

An investigation has been conducted in the Langley stability tunnel to determine the effects of changes in horizontal-tail location on static longitudinal stability and on steady-state rotary damping-in-pitch of a complete model with wing and tail surfaces having the quarter-chord lines swept back 45°.

Results of the investigation show that, at low angles of attack, although changes in vertical position of the horizontal tail relative to the wing have significant effects on static longitudinal stability, these changes have no significant effect on rotary damping-in-pitch. The standard methods of calculating the tail contribution to damping-in-pitch at low angles of attack are found to be reliable for all horizontal-tail locations investigated.

At high angles of attack, the static longitudinal stability characteristics are improved by moving the horizontal tail downward; whereas the rotary damping-in-pitch generally was increased by moving the horizontal tail upward.

From author's summary

4251. Garfinkel, B., Minimal problems in airplane performance, *Quart. appl. Math.* **9**, 2, 149–162, July 1951.

Author considers flight path of aircraft (considered as a particle), allowing for variation of fuel consumption and power, and derives three equations for the five generalized coordinates (horizontal and vertical position, mass, altitude, and power). The minimal problem (e.g., finding flight path for minimum time of flight, or maximum range) is then of the Bolza type in the calculus of variations. Euler-Lagrangian equations are derived and lead to a two-parameter family of curves, representing all possible solutions. Special cases discussed are (1) problems not involving the time, and (2) problems not involving the range.

The paper applies only to low trajectories and to the range of velocities in which drag coefficient can be expressed as $C_D = A + C^2_L/B$, where A, B are constants.

A. W. Babister, England

4252. Donegan, J. J., and Pearson, H. A., Matrix method of determining the longitudinal-stability coefficients and frequency response of an aircraft from transient flight data, *Nat. adv. Comm. Aero. tech. Note* 2370, 29 pp., June 1951.

A method is presented for calculating the longitudinal stability coefficients of an airplane from flight test data. Assumptions are made as to the form of the equation of motion of the airplane and, as a consequence of these assumptions, a number of constant coefficients for an airplane are defined. The equations of motion are then expressed in integral form, and the necessary integrations are carried out using test information. Authors point out that the undetermined coefficients may be evaluated if as many times are selected for the integration as there are undetermined coefficients. They suggest that more equations be formed than there are unknowns and the least-square method used for getting the best fit coefficients to the assumed equation of motions. Matrix methods are used to express the operations. Article shows how to use the evaluated coefficients in obtaining the frequency response functions for the airplane. Authors state that although the method is no more accurate than several other available methods, it requires less extensive instrumentation in the airplane at the expense of somewhat more computation.

John E. Stevens, USA

4253. Squire, H. B., Jet flow and its effects on aircraft, *Aircr. Engng.* **22**, 253, 62–67, Mar, 1950.

4254. Miele, A., Study of the jet-propelled airplane held in a steady, properly banked turn (in Italian), *Riv. aero.* **27**, 1 (*n.s.*), 23–35, 1951.

An exhaustive catalog of ideal turning performance exhibited by a plane executing a steady constant-altitude maneuver. Because of assumption usually made that thrust remains constant with speed for turbojets, relationships describing features of turns are expressible in capsule formulas. Three special turns are treated in pedantic minuteness: Cases of maximum curvature, maximum angular velocity, and maximum roll (though properly banked). Routine conclusions, in general, systematize or extend results applying to prop-driven planes, such as derived by Gates [*Rep. Mem.* no. 1502, Aug. 1932]. Over-all observation is that incidence angles for specified turns with prop-driven planes are always larger than for jet planes with same polars; also, incidences corresponding to three maximized turns have magnitudes standing in order of above listing, the smallest (maximum roll) being identical to that for maximum endurance in level flight for jets *and also* prop-driven planes. Of course, this is condition for maximum L/D for jets, but maximum $C_L^{3/2}/C_D$ for piston-engined types.

R. H. Cramer, USA

4255. Tomotika, S., Tamada, K., and Umemoto, H., The lift and moment acting on a circular-arc aerofoil in a stream bounded by a plane wall, *Quart. J. Mech. appl. Math.* **4**, part I, 1–22, Mar. 1951.

Expressions are derived for the lift and moment of a circular arc airfoil produced by a flow that is bounded by a plane wall. The numerical example carried out in the paper indicates that, as this airfoil approaches the wall, the lift and moment coefficients first decrease and then increase to values which are greater than the corresponding values for an arc airfoil in an unbounded stream. It is further shown that the camber of the airfoil influences the effect of the wall on the lift and moment coefficients of the airfoil.

Harry H. Hilton, USA

4256. Bismut, M., Rapid determination of the wave drag of sweptback wings (in French), *Rech. aéro.* no. 21, 9–17, May–June 1951.

Author considers the linearized supersonic flow past a sweptback wing consisting of two intersecting cones which extend forwards from the tips. To find an approximate value of the wave drag of the wing, the curved cross section of each cone is replaced by a polygon; the total flow field can then be determined by superposition of a finite number of conical fields. Results are computed most rapidly when, starting at the leading edge, the length of each side of the polygon increases in geometrical progression.

Maurice Holt, England

4257. Jacobs, W., Systematic six-component measurements on swept wings (in German), *Ing.-Arch.* **18**, 5, 344–362, 1950.

While the theory of swept wings has made great progress for supersonic flow, no equally satisfactory solution for subsonic, especially incompressible flow is available. A comprehensive report is given on test results which were obtained from 1941 to 1945 in a 1.2-m tunnel at the Aerodynamics Institute in Braunschweig. According to the author, those test results are no longer accessible. Trapezoidal wings with airfoil section NACA 23012 were tested at Reynolds number 4×10^5 by systematic variation of the sweep angle (30° forward, 0°, 15°, 30°, and 45° backward), taper ratio (tip to root chord 0.20, 0.60, 1.00), angle of attack, and angle of yaw. The stability derivatives for rolling and yawing moments show consistent variations and are in good agreement with the few curves which can be calculated by Multhopp's and Weissinger's theoretical approximations. Lift coefficient, drag coefficient, and lift-curve slope show the expected proportionality to $\cos^2 \beta$ where β is the angle of yaw; one surprising exception is the drag coefficient of the sweptforward wing which is practically independent of β.

G. R. Graetzer, USA

4258. Jacobs, W., Systematic measurements of pressure distribution on sweptback wings of constant depth for symmetric and unsymmetric incident flow (in German), *Ing.-Arch.* **19**, 2, 83–102, 1951.

Continuing the force tests [see preceding review], author reports chordwise and spanwise pressure-distribution tests on yawed sweptback wings made in 1942–1945. Wings of constant chord, NACA 23012, were tested under systematic variation of sweep

from 0° to 45°, yaw from 0 to 20°, and angle of attack from 1.9° to 14.3°. Wind speed 40 m/sec, Re = 420,000. A theory is developed which permits the approximate calculation of lift distribution on yawed wings with sweep; so far, only theories for yawed straight wings have been available. The theory is in reasonable agreement with test results, except for the aftermost wing panel at large angles of attack. All wings show a small region of decreased local pressure coefficient at the center section; the decrease becomes more pronounced at higher angles of attack. It is further evident that the foremost wing panel of the sweptback wing has more lift than the aftermost wing panel: the difference increases with angle of sweep. However, the foremost wing panel of the straight wing has slightly less lift than the aftermost wing panel; local tip effects show the opposite behavior. G. R. Graetzer, USA

4259. Taylor, J. L., An analysis of the lift on straight, yawed and sweptback wings, *Aero. Quart.* **2,** part 4, 293–304, Feb. 1951.

The downwash of an arbitrary wing is analyzed. The following effects may be summarized: (1) Finite-chord effect: This is calculated for a straight rectangular wing. The downwash is composed of the downwash used in the lifting-line theory and of the simple downwash correction. (2) Yaw effect: For infinite aspect ratio, the spanwise and chordwise downwash distribution can be reduced by the cosine theorem. The correction of the cosine factor due to the finite aspect ratio is fairly small and can be neglected. (3) Tip effect and kink effect: The influence of leading and rear tips for yawed wings and that of the central kink and rear tip for swept wings is locally limited and of universal character. N. Scholz, Germany

4260. Smaus, L. H., and Stewart, E. C., Practical methods of calculation involved in the experimental study of an autopilot and the autopilot-aircraft combination, *Nat. adv. Comm. Aero. tech. Note* 2373, 26 pp., June 1951.

Detailed formulas for relating the open and closed loop transfer functions of an autopilot-controlled aircraft are derived. Displacement plus rate-of-displacement errors are considered, with feedback of angular displacement and rate of angular displacement. Comparison with experiments is given. Andrew Vazsonyi, USA

4261. Bleviss, Z. O., Some roll characteristics of cruciform delta wings at supersonic speeds, *J. aero. sci.* **18,** 5, 289–297, May 1951.

A qualitative discussion of three roll problems associated with a supersonic missile having a circular cross section and a cruciform delta wing arranged on a practically cylindrical portion of the body. Problems considered are: (1) Roll due to aileron deflection of two opposite fins, (2) damping in roll, and (3) roll induced by pitch and yaw. Some theoretical results for simple schemes, most of which are based on nonviscous linear theory, are presented but not derived. Limitations of stationary, nonviscous linear theory are pointed out for various roll problems: large angles of pitch and yaw, nonstationary roll-damping effects, wing-body-tail interference. Finally, some recommendations are made for future theoretical and experimental research. Pierre Schwaar, Switzerland

4262. Letko, W., Effect of vertical-tail area and length on the yawing stability characteristics of a model having a 45° sweptback wing, *Nat. adv. Comm. Aero. tech. Note* 2358, 49 pp., May 1951.

Results indicate agreement between the estimated and the measured contributions of the tail to yawing derivatives. The interference between the various model components was found to be small. A. Petroff, USA

4263. Falkner, V. M., Rotary derivatives in yaw. Calculation by lifting plane theory of the rolling and yawing moments of a wing due to rotary motion in yaw, *Aircr. Engng.* **23,** 264, 44–50, 54, Feb. 1951.

The case studied is that of a wing moving in a circular path, the axis of the circle being normal to the plane of the wing. Calculations are carried out for two elliptic wings, with ratios of major to minor axis 2.5 and 5 to 1. The distribution of lift and the resulting yawing and rolling moments are calculated by the lifting surface method previously developed by the author [Falkner, V. M., "The calculation of aerodynamic loading on surface of any shape," *Rep. and Mem.* 1910; Falkner, V. M., and Lehrian, D. E., "Calculated loadings due to incidence of a number of straight and sweptback wings," *Rep. and Mem.* 2596].

The principal factors considered in this case are the variation of velocity across the span, and the curvature of the wake. Special downwash tables are presented for curved horseshoe vortexes in order to take care of the latter factor. For comparison, lifting-line solutions are given with the wake curvature both included and neglected. The difference in the derivatives is of the order of 10%.

It is shown that applying the actual velocity distribution to the C_l variation obtained for straight flight (a very simple approximation) leads to values of C_{l_r} about 40% too low. B. Etkin, Canada

Aeroelasticity (Flutter, Divergence, etc.)

4264. Williams, J., Aircraft flutter, *Aero. Res. Counc. Lond. Rep. Mem.* 2492, 62 pp., 1951.

Report sums up available knowledge of aircraft flutter up to year 1948, based mainly on British research. It is a good summary but contains little that is new to American workers in this field. Benjamin Smilg, USA

4265. Duncan, W. J., The fundamentals of flutter, *Aero. Res. Counc. Lond. Rep. Mem.* 2417, 36 pp., Nov. 1948, published 1951.

Paper, intended primarily as an introduction to flutter for the nonexpert reader, first outlines in nonmathematical language the physical mechanism of bending-torsion flutter. This discussion contains a description of an ingenious device called the "flutter engine" which illustrates how a fluttering wing extracts energy from the airstream. Paper then gives differential equations appropriate to two-degree-of-freedom flutter and discusses their solution for a critical flutter speed. Although author's mathematical formulations and terminology are peculiar to the British school of flutter, reviewer believes that the paper would serve as a good introduction to flutter for readers everywhere. Bernard Budiansky, USA

4266. Runyan, H. L., Single-degree-of-freedom-flutter calculations for a wing in subsonic potential flow and comparison with an experiment, *Nat. adv. Comm. Aero. tech. Note* 2396, 27 pp., July 1951.

Using background of flutter theory developed in references, curves are presented of two-dimensional wing flutter-pitching parameters for various Mach numbers and moment pivot points. Curves indicate Mach effects are strong. This type flutter is demonstrated experimentally; the theory is in good agreement for high inertia-parameter values. John DeYoung, USA

4267. Engelbrecht, A. E., Coupled free vibrations of a swept wing, *J. aero. Sci.* **18,** 5, 329–338, May 1951.

In the trial-frequency method of this paper, the continuous elastic structure is approximated by a finite number of concentrated masses connected by weightless elastic sections. The method is similar to that developed by the reviewer ["Vibration analysis," New York, McGraw-Hill Book Co., 1944] but is extended to take care of cases where the elastic axis has a circular planform. Also, the tabular calculation used by reviewer is replaced by a series of matrix multiplications, which unfortunately results in a considerable increase in the amount of numerical work.

N. O. Myklestad, USA

4268. Rea, J. B., Dynamic analysis of aeroelastic aircraft by the transfer-function Fourier method, *J. aero. Sci.* **18,** 6, 375–397, June 1951.

Paper outlines the method of applying standard servomechanism transfer-function analysis procedures to problems in airplane dynamics. The well-known transfer-function concept is reviewed together with the Fourier technique for determining transient response. Application of the method to a number of practical examples is described in somewhat general terms. These examples are (a) determination of transient tail loads developed by elevator displacement, (b) stability and performance analysis of an airplane with a gust-load alleviation system, and (c) a combined analysis of flight-path stability, dynamic tail loads, and flutter response of control system and structure.

Dana Young, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 4155, 4204, 4205, 4268, 4284)

4269. Slivka, W. R., and Silvern, D. H., Analytical evaluation of aerodynamic characteristics of turbines with nontwisted rotor blades, *Nat. adv. Comm. Aero. tech. Note* 2365, 52 pp., May 1951.

On the assumptions that gas-turbine blades should incorporate cooling passages, that the manufacture of twisted and cooled stator blades will be relatively easy because of the freedom in selecting materials and methods that the low stator-blade stress provides, whereas rotor blades to be cooled should be nontwisted for manufacturing simplicity, and that high efficiency requires constant relative rotor-entrance flow angle along the radii of nontwisted rotor blades, a method was developed for calculating the radial variation of stator-exit angle required to obtain constant relative rotor-entrance angle. Then the aerodynamic characteristics were calculated and compared with those for free-vortex rotors over the range of current design limitations, and no significant differences were found. The effect on performance of the radial variation in stator-flow conditions is not known, but greater variations are used in axial compressors with no decrease in efficiency. Large-scale design charts are included.

Benjamin Miller, USA

4270. Jarre, G., Flow of a compressible fluid through a radial flow turbomachine impeller (in Italian), *Termotecnica* **5,** 2, 77–81, Feb. 1951.

General equations for the frictionless absolute flow through a radial runner are established, assuming variable fluid density. These equations are linearized by neglecting influence of relative velocity change on enthalpy variation through the impeller, thus taking only centrifugal effects into account. A solution of the linearized equations is found, based on the Betz-Ackeret method of expressing the bound vortexes of the blading by means of Fourier series. The solution is a generalization of results found by Betz for incompressible flow.

Reviewer feels that the practical value of this theory is rather limited due to preponderant influence of friction in radial runners.

E. Haenni, Switzerland

4271. Sinnette, J. T., Jr., and Costello, G. R., Possible application of blade boundary-layer control to improvement of design and off-design performance of axial-flow turbomachines, *Nat. adv. Comm. Aero. tech. Note* 2371, 32 pp., May 1951.

Gains that may be expected in the stage-pressure ratio and efficiency of axial-flow compressors when boundary-layer control by suction or ejection is applied are discussed. Suction is recommended in the later stages of compressors and ejection for cooling and simultaneously increasing blade loading on turbine stages. Conformal mapping is used to design suction or ejection-slotted blades with a prescribed velocity distribution around the blade and in the slot. The method applies for potential flow of a compressible fluid with a linear pressure-volume relationship.

E. R. G. Eckert, USA

4272. Dubs, R., On the runaway of hydraulic turbines (in German), *Bull. Assn. suisse Elect.* **42,** 11, 377–384, June 1951.

Author investigates the speed-time relation of turbines when suddenly unloaded, the gate opening remaining constant due to failure of the governor mechanism. He adopts a torque-speed relation for constant head and gate opening of the type $M = a - bn - cn^2$ and states that many experiments have verified its suitability for all types of turbine. The coefficients are obtained from known values at normal operating conditions, at zero speed, and at zero torque. Windage resistance of the generator is allowed for by a quadratic term, and the differential equation for the acceleration of the unit is integrated to give the speed-time relation.

Author describes experiments carried out at the E.T.H., Zurich, on a Pelton wheel, a Francis turbine, and a Kaplan turbine, to check the theoretical results. To obtain accurate observations with rapid speed changes, the tachometer and a precision stop watch were photographed simultaneously with a 16-mm cine camera. Satisfactory agreement between theoretical and experimental speed-time curves was obtained.

Author points out that assumption of constant head is not strictly correct, and that, particularly in Kaplan installations, the decreased head due to increased flow under runaway conditions should be allowed for.

Alan Burn, Australia

4273. Legendre, R., Various remarks on the flow in turbomachines with variable circulation (in French), *Bull. Assn. tech. marit. aéro.* no. 49, 815–843, 1950.

This analytical study of the three-dimensional flow of a perfect fluid (both incompressible and compressible) in axial turbomachinery consists essentially of an attempt to extend Prandtl finite wing theory to this difficult problem. Rotation symmetry is not assumed a priori, but author defines a mean giratory flow of velocity components A, B, C in axial, radial, and peripherical directions:

$$A.\delta\alpha = \int u.d\alpha \quad B.\delta\alpha = \int v.d\alpha \quad C.\delta\alpha = \int w.d\alpha$$

where $u(x, r, \alpha)$, $v(x, r, \alpha)$, $w(x, r, \alpha)$ are the actual velocity components and the integrals are taken between two adjacent Prandtl discontinuity surfaces at constant x and r. This flow depends only on x and r, and author establishes first its general properties. He then separates treatment of the flow after, within, and before an isolated rotor or stator blade row, using systematically Stokes' stream function ψ. Solutions of these

partial problems are indeterminate and must be brought together by adequate connecting conditions. This is achieved for the inverse cascade problem, where velocity distributions are given before and after the blade row, while the direct cascade problem appears to be intractable. Solutions are expanded in series and the joining conditions are imposed to ψ and ψ_x for a simple interpolation of the flow within the cascade. It is possible to realize the continuity of ψ_{xx} and ψ_{xxx} (curvature), but this does not seem to be justifiable considering the simplifications already made (the blades are replaced by their skeleton).

Last part of memorandum deals with the approximate treatment of a stator or rotor blade row placed downstream of a rotating or fixed one. This problem is much more complicated because the flow relative to the subsequent cascade is not permanent. This difficulty cannot be taken into account and the entering flow is assumed to be the limit mean giratory flow downstream of the precedent blade row. The problem is then treated essentially in the same manner. Paper ends with some qualitative remarks on blade design, the influence of the blade wakes and boundary layers, the interaction of closely spaced cascades, etc. An example of compressor stator and rotor cascade is shown in two appendixes without any comment.

Presentation is concise and elegant. Some typographical errors and omissions occur, and the fluid mechanician interested in basic flow problem of turbomachinery will compare Legendre's attempt with Siestrunck's and Fabri's work [AMR **4**, Rev. 3031] with great profit. Pierre Schwaar, Switzerland

4274. **Luskin, H., and Klein, H., High-speed aerodynamic problems of turbojet installations,** *Trans. Amer. Soc. mech. Engrs.* **73**, 4, 375–384, May 1951.

Examination of the flight-speed trend shows that supersonic airplanes with turbojet engines are to be expected soon. Main difficulty: For same ratio of power-plant weight to gross weight, thrust/weight ratio of supersonic power plant must be 3 to 4 times the subsonically acceptable value because of supersonic reduction of lift/drag ratio. Following a short discussion of afterburning (an afterburner increasing static thrust 50% will yield, for the same peak temperature, tripled thrust at Mach number 2.0), paper deals with items influencing engine performance which are within control of the airframe designer. Inlet stagnation-pressure losses are shown to be particularly important at high speeds and for engines with low component efficiencies, low turbine-inlet temperature, high pressure ratio, and without afterburning. Outlet-nozzle losses also become increasingly important at high speeds. Incremental scoop drag, effect of airplane speed on inlet-engine matching, outlet-nozzle losses, and cooling drag are discussed to demonstrate necessity for adjustable inlet and outlet. Remainder of paper deals with flight-speed stability and jet-tail interference. It is suggested that tendency of turbojet airplanes toward speed instability, resulting from parallelism of thrust and drag curves vs. speed, be remedied by some form of automatic transient thrust-variation control. Joseph V. Foa, USA

4275. **Roy, M., Adaption of jets to supersonic aircraft propulsion** (in French), *Off. Nat. Étud. Rech. aéro. Note tech.* 1, 18 pp., 1950.

Superiority of turbojet over ramjet is demonstrated through simplified performance analysis and discussion of basic criteria for power-plant selection, with particular attention to supersonic flight applications. Typical calculated over-all efficiencies at flight Mach number 1.4 are about 35% for turbojet with turbine-inlet temperature 1850R, 30% for turbojet with afterburner and peak temperature 2300R, and 16% for ramjet with same peak temperature. Corresponding thrust per unit maximum cross-sectional area of turbojet without afterburner (assuming that burners occupy only 60% of maximum cross section) is over five times that of ramjet with burner occupying entire cross section. Except for flights lasting only a few minutes, weight difference of fuel consumed by ramjet and turbojet for same thrust exceeds weight difference of the two power plants, thereby offsetting weight advantage of ramjet. It is also stated that claims of ramjet superiority from standpoints of simplicity and cost may be refuted when consideration is given to necessity for launching means and auxiliary equipment. Joseph V. Foa, USA

4276. **Jakobsson, B., Definition and measurement of jet engine thrust,** *J. roy. aero. Soc.* **55**, 484, 226–243, Apr. 1951.

"Front external" (ahead of the intake section), "internal," and "rear external" (aft of the nozzle exit) thrusts are defined by the author. "Total thrust" is the sum of the three and it is claimed that this total thrust does not act on the engine mountings. Standard static thrust tests (on test beds), as well as thrust measurements in flight, are described briefly. Tests in altitude chambers are also described. Author proposes a new static test method with various extensions mounted aft of the exit nozzle. Evaluation of the thrust in various flight conditions from the static test data is based on "equivalent" temperature measurements in flight and at the test bed. To quote: "There are some questionable points in the suggested method." The author invites criticism of his suggestions. Paul Torda, USA

Flow and Flight Test Techniques

(See also Revs. 4198, 4257, 4258)

4277. **Hensel, R. W., Rectangular wind-tunnel blocking corrections using the velocity-ratio method,** *Nat. adv. Comm. Aero. tech. Note* 2372, 40 pp., June 1951.

Ratios of velocity increments at test bodies to those at wind-tunnel walls caused by blocking are presented. The method furnishes semi-empirical blocking correction for models in closed rectangular wind tunnel.

From author's summary by A. Petroff, USA

4278. **Klunker, E. B., and Harder, K. C., On the second-order tunnel-wall-constriction corrections in two-dimensional compressible flow,** *Nat. adv. Comm. Aero. tech. Note* 2350, 23 pp., Apr. 1951.

First- and second-order tunnel-wall induced velocities on symmetric parabolic arc airfoils at zero angle of attack in a closed wind-tunnel are calculated by the Prandtl-Busemann small-disturbance iteration method. The distribution of induced velocity along the chord is presented at a few Mach numbers for two ratios of tunnel height to chord length. Also included is the variation with Mach number of the tunnel-wall induced velocity at the chord midpoint for two airfoil thicknesses and three ratios of tunnel height to chord length. Neal Tetervin, USA

4279. **Hufton, P. A., Cook, F. G. R., and Saunders, P. S., A recording system for flight test data,** *Aero. Res. Counc. Lond. curr. Pap.* 44, 7 pp., 7 figs., Dec. 1949, published 1950.

A recording system for quasistatic flight-test data enabling automatic recording, reading, and analysis of data is proposed. Authors devised method to obtain electrical signal in binary notation for digital recording from the movement of standard aircraft instruments of the rotary type. Inadequate torque of aircraft instruments is overcome by interposing a pulse servomotor which follows indicator and drives a computator of five-channel capacity with contacts graduated to yield a signal in binary notation.

Two five-channel units are so connected that a resolution of 0.1% full-scale deflection is obtained.

A transmitter, small enough to be enclosed with indicator in sealed cases, has been built.

When this paper was written, the system had not been sufficiently developed for use in flight tests.

Maurice A. Walter, USA

4280. Dhawan, S., and Roshko, A., A flexible nozzle for a small supersonic wind tunnel, *J. aero. Sci.* **18,** 4, 253–258, Apr. 1951.

Paper describes in some detail the design, fabrication, and operation of a flexible nozzle for a small supersonic wind tunnel (4 by 10 in.) of moderate operating range ($M = 1.1$ to 1.5). Continuous variation in nozzle shape is obtained through use of a flexible wall of nonuniform thickness in combination with a simple system of two screw jacks. Pressure surveys and schlieren and interferometer studies show the resulting flow to be sufficiently uniform throughout the operating range. Paper can be recommended to anyone contemplating design of a small supersonic tunnel.

Walter G. Vincenti, USA

4281. Bonell, T., A simple pressure-time measuring element for ballistic purposes (in Swedish), *Tekn. Tidskr.* **80,** 27, p. 659, Aug. 1950.

Principle of element is resistance change of graphite when subjected to varying pressure. Unit consists of steel bushing containing pressure-sensitive element, isolated from bushing by heavy oil or grease. Pressure is recorded through electrical setup of bridge, amplifier, and oscillograph. Paper claims that element is superior to piezo method regarding time consumed per measurement, good experimental agreement with piezo method, and versatility in use, covering pressure-time histories ranging from a few millisec to several minutes. Examples of use are pressure measurements on small arms and thrust measurements on rockets.

T. A. Mortensen, USA

4282. Ball, J. G., and Adams, E. T., A method of temperature programme control for thermal analysis, *J. sci. Instrum.* **28,** 2, 47–49, Feb. 1951.

A method is described for controlling the rate of heating or cooling of small systems of essentially constant specific heat. This is achieved by a simple but unique cam-and-spindle arrangement which varies the rate of angular rotation of the shaft of a variable transformer, thus changing its voltage output which is fed to the system whose rate of change of temperature is to be controlled. Complete analysis is presented for the design of the cam for various heating or cooling rates. Calibration is required to determine the steady-state temperature-voltage characteristics of any particular furnace-transformer combination. Experimental results are given for the heating and cooling of a 0.05-cc aluminum specimen which was programmed to cool at 2 C per min and which actually cooled at 2.2 C per min.

John A. Clark, USA

4283. Naeser, G., and Pepperhoff, W., Optical temperature measurements of luminous flames (in German), *Arch. Eisenhüttenw.* **22,** 1/2, 9–14, Jan./Feb. 1951.

Paper discusses optical characteristics of luminous hydrocarbon flames; calculation of flame temperature from color and black body radiation; relationship between wave length and absorption coefficient of luminous flames; size of carbon particles in luminous flames; measurement of flame temperature by means of the "BIOPTIX" color brightness pyrometer.

From authors' summary by Antoni K. Oppenheim, USA

4284. Lalive d'Epinay, J., Aerodynamic methods applied to turbo-machine research, *Brown Boveri Rev.* **37,** 10, 357–367, Oct. 1950.

Author describes new test equipment and discusses some of the fundamental questions posed by the fluid flow in turbomachines, especially with regard to the axial compressor.

From author's summary

4285. Fusfeld, R. D., A probe for measuring flow inclination in a supersonic air stream, *J. aero. sci.* **18,** 5, p. 356, May 1951.

Probe described is a form of blunt body specially designed for use in supersonic flow. From author's summary

4286. Kane, E. D., Sphere drag data at supersonic speeds and low Reynolds numbers, *J. aero. Sci.* **18,** 4, 259–270, Apr. 1951.

Measurements of drag forces on spheres (0.10 to 1.00-in. diam) have been made in a low density supersonic wind tunnel (Mach numbers from 2.1 to 2.8; Reynolds numbers from 15 to 800). The drag coefficient increased with increasing Reynolds number by a factor 2.5 over the test range, whereas available information at supersonic speeds for the same Mach numbers but a Reynolds numbers of $R \cong 10^5$ could be correlated as a function of the Mach number alone. An equation $K_D = f(R_1)$ is given. Flow visualization photographs indicate a possibility that interaction occurred between the sphere boundary layer and shock wave at low Reynolds numbers.

From author's summary by B. Regenscheit, Germany

Thermodynamics

(See also Rev. 4283)

4287. Schmidt, E., The total-heat, air-ratio diagram: A new method for the calculation of gas-turbine cycles, *Proc. Instn. mech. Engrs.* **161** (*W.E.P.* no. 52), 203–211, 1949.

Author proposes diagram of enthalpy vs. equivalence air ratio λ (ratio of actual air supplied to air for stoichiometric combustion) as a tool in the analysis of gas-turbine cycles. One diagram must be made for each fuel. Enthalpies are for quantity of air supplied per unit mass of fuel, or for quantity of product gas produced per unit mass of fuel. Two sets of isotherms appear on diagram, one set for air and the other for products. Both sets are straight lines. Once fuel, pressure ratio, compressor and turbine efficiencies, and inlet temperature are assigned, a simple graphical construction leads to a clear portrayal of out-put power and cycle efficiency for all λ, or air : fuel ratios. For open cycle with heat exchanger, graphical construction carried out for fixed turbine-inlet temperature leads to output power and cycle efficiency as a function of heat-exchanger effectiveness. Variable specific heats and actual properties of combustion gases are considered. Dissociation and pressure effects on specific heats are neglected. Auxiliary entropy diagram facilitates calculation of temperature changes across compressor and turbine. Stewart Way, USA

4288. Omori, T. T., and Orning, A. A., Effect of pressure on the combustion of pulverized coal, *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 591–597, July 1950.

Authors examine the theoretical relations for heat transfer to burning particles and reach the conclusion that increasing gas pressure affects adversely the ignition of pulverized coal in a furnace. An experimental investigation of this phenomenon was undertaken wherein furnace radiation temperature, gas pressure, and coal sample were varied. The data lead to the conclusion that decreased initial heating rates at higher pressures reduce efficiency of combustion. Increasing the radiation temperature,

however, produces more complete combustion, as evidenced by analysis of the ash. Increasing pressure and corresponding mass flow of gas through furnace may result in higher convection heat transfer, to the detriment of complete combustion (Discussion).

H. M. Spivack, USA

4289. Friedman, R., and Burke, E., On the one-dimensional theory of flame structure, *J. aero. Sci.* **18**, 4, 239–246, Apr. 1951.

Paper is restricted to cases where (a) heat flow is assumed to limit flame-propagation rate and diffusion is ignored, or (b) diffusion of a single species is assumed to limit flame propagation, heat flow, and diffusion of other species being ignored. Treatment differs from previous ones [e.g., AMR **4**, Revs. 414, 415] in that rate of heat release or of active species formation is arbitrarily assumed as given function of coordinate x normal to flame front. This enables integration of equation of heat balance or of particle conservation in closed form. Results for variety of heat-release (or species formation) patterns are presented in tabular and graphical form. Comparison with experiment is made only for case of a low-pressure acetylene-oxygen flame [AMR **3**, Rev. 325]. Authors claim that, when more precise measurements on flames are available, relations developed in paper will aid understanding of combustion mechanism. In appendix, approximate relations between local Mach number of flame, mean free path, flame thickness, and reaction probability are derived.

George H. Markstein, USA

4290. Mordell, D. L., The exhaust-heated gas-turbine cycle, *Trans. Amer. Soc. mech. Engrs.* **72**, 3, 323–329, Apr. 1950).

As an alternative to the gas-turbine regenerative cycle, fuel may be burned in the stream approaching the heat exchanger from the turbine, rather than in the stream leaving the heat exchanger and going to the turbine. This may be particularly advantageous when it is desired to keep fly ash or other erosive combustion products out of the turbine. The turbine runs on hot air in this arrangement.

Authors have investigated this exhaust-heated cycle. For given compression ratio, turbine-inlet temperature, and heat-exchanger-thermal ratio, the cycle efficiency is lower with the exhaust-heated cycle than with the conventional open cycle with heat exchanger. However, with compressor intercooling and turbine interstage reheating, it equals or exceeds that of the conventional regenerative open cycle. With steam generation by the final exhaust stream, a further very large increase in cycle efficiency may be realized, reaching finally 36% for 4.5 to 1 compression ratio and 1400° turbine-inlet temperature.

Consideration is given to differences in the combustion conditions in the exhaust-heated and the conventional cycle. The pressure level is here low, and good mixing with the dilution air is less important. The heat-exchanger and control problems are also briefly discussed. Finally, some aspects of practical application as for power generation, rail, or marine service are considered.

Stewart Way, USA

4291. Prigogine, I., and Mahieu, M., On the perturbation of the Maxwell distribution by chemical reactions in gas phase, *Physica* **16**, 1, 51–64, Jan. 1950.

This is the second of a series devoted to the extension of Chapman-Enskog's method to inelastic collisions [AMR **4**, Rev. 896]. This note is concerned mainly with the study of the influence of the heat of reaction on the Maxwell distribution. As shown in the first paper this effect is, in general, small for the activation energy. The heat of reaction, however, can perturb the Maxwell distribution to an appreciable extent. As a result, the reaction rate is increased for exothermal, decreased for endothermal reactions. The effect is especially important for exothermal reactions. At the same time, the order of the reaction is increased.

Calculations have been performed for the initial rate of a reaction of the type $A_0 + B \rightarrow A_1 + B$. In this case the result can be expressed by the following equation:

$$v = v^{(0)}\,(1 + 1.2\, x_{A_0} x_B \nu_{VT}/\epsilon^*)$$

where ϵ^* is the activation energy, ν_{VT} the heat of reaction and x_{A_0}, x_B the mole fractions of A_0 and B; $v^{(0)}$ is the reaction rate estimated by assuming Maxwell distributions for each constituent.

From authors' summary

4292. Burkhardt, G., Theoretical contribution to the paper by R. Hilsch on the "vortex tube" (in German), *Z. Naturforsch.* **3a**, 46–51, Jan. 1948.

Author makes certain simplifying assumptions to permit a rather rough flow and thermodynamic analysis of the Hilsch vortex tube. Analysis disregards entirely details of internal flow and heat transfer within the vortex. Assumptions relate to certain simplified mathematical forms which are used to describe relations of certain of the flow variables (such as relation of pressure ratio across outlet nozzle and fraction γ passing out cold outlet). The treatment presupposes an internal transfer of heat from one part of the stream to another, and thus assumes rather than explains the basic mechanism.

Within these limitations, author's analysis does throw some light on performance of the vortex tube. Performance coefficient is about 0.2 to 0.3 at optimum conditions for vortex-tube refrigerator as compared to 4 or 5 for vapor-compression machine, or 1.0 for gas-cycle refrigeration.

Stewart Way, USA

4293. Campbell, A. S., Thermodynamic properties of reactive gas mixtures, *J. Franklin Inst.* **251**, 4, 437–452, Apr. 1951.

Author develops formulations which show how much the thermodynamic properties of mixtures of reactive but otherwise ideal gases differ from corresponding values for mixtures of chemically inert ideal gases. Method is illustrated by use of a four-component mixture containing two reagents and two products. Expressions for heat capacity, enthalpy, entropy, extent of reaction, compressibility, and thermal coefficient of expansion are derived. For certain systems, e.g., one in which the heat of reaction is zero, various expressions reduce to inert gas forms.

In deriving above results, author defines a quantity Ψ as a function of stoichiometric coefficients and mole fractions of components, plus the change in number of moles caused by the reaction. Later, Ψ is related to the chemical mobility of the system; i.e., the amount of reaction caused by a given stress applied to a system in equilibrium, which is always positive. Hence, the heat capacity of a mobile or reactive system is always greater than that of a comparable inert system.

Systems of reactive gases may be divided into three classes, depending upon whether the product of heat of reaction times change in number of moles is (1) zero, (2) positive, or (3) negative. The latter class appears to be unique in that on a pressure-volume chart the lines of adiabatic and isentropic expansions of separate systems cross at five points rather than at only one. Author recognizes that this result may not be widely applicable, since it is valid only for mixtures in which all gases are nearly perfect. However, he infers that it might apply to the ozone-oxygen system.

The study of more general systems, e.g., systems with both inert and reactive gases or systems involving more than one reaction, would require a great extension of this analysis.

Reviewer doubts the engineering utility of the limited analysis. In addition to restrictions listed by author, any suitable reaction

would have to be sufficiently rapid to shift the composition as fast as conditions change. Therefore, author's choice of an ozone-oxygen mixture as illustrative of a Class 3 reaction is somewhat unfortunate. The equilibrium concentration of ozone is infinitesimal at absolute zero and decreases as temperature rises. A shift in equilibrium composition would have no detectable thermodynamic effect under attainable conditions. Reviewer suspects the same limitation holds for all Class 3 reactions. Jack D. Bush, USA

4294. Watanabe, T., On a method for calculating the volumetric efficiency of the internal combustion engine, *Memo. Fac. Sci. Engng. Waseda Univ., Tokyo,* no. 14, 12–13, 1950.

An equation is derived for the volumetric efficiency based on certain assumptions as to the type of flow into the engine cylinder. The resulting equation is in terms of quantities that can be measured. A. O. Flinner, USA

4295. Levedahl, W. J., and Howard, F. L., An apparatus for studying autoignition of engine fuels: results with normal heptane and normal hexane, *J. Res. nat. Bur. Stands.* **46,** 4, 301–309, Apr. 1951.

Authors succeed in demonstrating that several combustion phenomena such as two-stage combustion reactions, gas ionization in flames, and reaction catalysis by preconditioned walls and residual gases can be detected in auto-ignition engine cycles. Such information is usually obtained in simple combustion tube and bomb experiments. Authors found the first-stage combustion reaction to be relatively unaffected by mixture composition. The optimum conditions for the initiation of the second-stage reaction occurred at a stoichiometric mixture composition and became progressively worse as the mixture was made linear. No gas ionization was detected in the first-stage reaction. The second-stage reaction ionization was greatest for a stoichiometric mixture composition.

Modifications of ASTM-CFR fuel testing engine to operate at various fuel-air ratios on prevaporized premixed fuel of known composition are discussed. Equipment for measuring and automatically recording cylinder pressure and gas ionization in terms of crank angle are also described. Authors also describe a device for limiting cylinder pressure during a cylinder-flushing run of the engine. Thomas P. Clark, USA

Heat and Mass Transfer

(See also Revs. 4117, 4196, 4207, 4288, 4328, 4346, 4348)

4296. Hausen, H., Heat transfer in counter flow, parallel flow and cross flow [Wärmeübertragung im Gegenstrom, Gleichstrom und Kreuzstrom], Berlin, Springer-Verlag, 1950, xii + 464 pp. DM 69.

Book is divided into three parts: The first chapter deals with the heat-transfer coefficient and the pressure drop in pipes and channels; the second part contains problems of "recuperators" or heat exchangers with separate space for the flow of hot and cold fluids; while the third chapter is concerned with "regenerators" or heat exchangers where the heat is stored in walls of a chamber passed by hot and cold fluids in turn. As indicated by the title, author's emphasis is put on the second and third parts. Starting from a wide variety of geometrical configurations, which are first described from the point of view of technical applications, mathematical problems arising are treated in great detail. Results of these usually voluminous calculations are presented in technically useful form.

The first part on the heat-transfer coefficient gives an introduction to the physics of the subsequent problems of the more formal mathematical kind. The material gathered in the latter field is remarkable, both from technical and theoretical points of view, and includes, also, many original contributions by the author. The book may be recommended, in accordance with its intentions, as a reference book on heat exchangers; for study purposes, though, a simpler presentation of fewer typical examples would be preferable. N. Rott, USA

4297. Coyle, M. B., An air-flow analogy for the solution of transient heat conduction problems, *Brit. J. appl. Phys.* 2, 1, 12–17, Jan. 1951.

A hydraulic analogy for solving transient heat-flow problems has been developed in which air pressure corresponds to temperature and mass of air to quantity of heat. Capillary tubing connects air reservoirs, effective volumes of which are controlled by liquid levels which also indicate air pressure. The particular feature of the apparatus is that the reservoirs may be constructed with variable cross section so that variable specific heat and thermal conductivity as a function of temperature may be taken into account. Reviewer feels that value of the analogy is mainly for demonstration purposes, because its unwieldy nature in a complex system makes it less desirable than the electrical analogy for problem solution, and simple systems are readily solved by numerical methods. Paul R. Trumpler, USA

4298. Mattarolo, L., Method of determination of thermal diffusivity of poor heat conductors (in Italian), *Nuovo Cim.* (9), 7, 5, 809–815, Oct. 1950.

Measuring the temporal variation at the center and at an arbitrary internal point for bodies of simple geometric shapes cooling in an ambient atmosphere provides, with the aid of a theorem by Duhamel [Carslaw and Jaeger, "Conduction of heat in solids," p. 204], a method for determining thermal diffusivity. M. J. Goglia, USA

4299. Peck, R. E., Fagan, W. S., and Werlein, P. P., Heat transfer through gases at low pressures, *Trans. Amer. Soc. mech. Engrs.* **73,** 3, 281–286, Apr. 1951.

The heat transfer between two parallel vertical plates was determined experimentally. The space between the plates was filled with a gas at low pressure, and the boundaries of this layer were enclosed by plate glass which was painted with glyptal lacquer to prevent transmission by radiation. The variation of this heat transfer with pressure, temperature difference, type of gas, and thickness of the gas layer were determined. When correlated the result was: Helium: $Nu = 8.0Gr^{0.04}(L/H)^{0.75}$, for $10^{-2} < Gr < 5(10)^3$; $h = 9.2kP^{0.08}$, $1 < P < 760$ mm Hg. Air: $Nu = 14.1Gr^{0.022}(L/H)^{0.75}$, for $10^{-1} < Gr < 5(10)^3$; $h = 17.1kP^{0.083}$, $1 < P < 100$ mm Hg.

Equipment used was designed for use at very low pressures, and was calibrated at "ultimate vacuum" where convection does not play any part. The losses due to convection (and conduction) should increase with increase in pressure, and authors obtain considerably higher values of heat transfer than have been observed by other investigators. It was necessary to write different dimensionless equations for helium and air; reviewer believes that additional properties of the gases should be used to enable correlating all results with one equation. R. M. Wingren, USA

4300. Ikenberry, E., The conservation of systems in phase space, *Quart. appl. Math.* **9,** 2, 195–203, July 1951.

Author derives the well-known transport equation for fluids from Gibbs principle of conservation of density-in-phase, assum-

ing no net flow of systems through the boundaries in phase space. Result is a special case of Enskog's transport equation in that the collision term is absent. This is of no consequence for liquids, however, and the hydrodynamical equations of motion that follow apply to both liquids and gases. Robert E. Street, USA

4301. Mann, W. R., and Wolf, F., Heat transfer between solids and gases under nonlinear boundary conditions, *Quart. appl. Math.* 9, 2, 163–184, July 1951.

In the theory of heat transfer between solids and gases the boundary condition usually is of the following form: $k(\partial U/\partial n) = -f\Delta U \ldots$ [1], where k is the thermal conductivity of the solid, $\partial U/\partial n$ is the thermal gradient at the surface, ΔU is the difference in temperature between the surface and the gas, f is the factor of proportionality, known as film-transfer factor. If the film-transfer factor f is a constant, the boundary condition [1] is linear. In general, f is a function of temperature and the boundary condition [1] is nonlinear.

Paper considers the problem of heat transfer of a semi-infinite body with f as an arbitrary function of surface temperature. The problem has been formulated in terms of a nonlinear integral equation for the temperature distribution at $x = 0$, i.e., $U(0, t)$; x is spatial coordinate and t is time.

First, the integral equation of surface temperature for the linear case is discussed. Then the nonlinear integral equation of surface temperature is thoroughly studied. Authors show that the method of successive approximations can be used to solve the nonlinear case. S. I. Pai, USA

4302. Radok, J. R. M., Solution of a heat flow problem, *Austral. J. sci. Res. Ser. A*, **4**, 1, 12–15, Mar. 1951.

Paper deals with the heat flow through a rectangle subject to the following boundary conditions: One end is completely insulated (without heat flux across it) and at the opposite end a constant temperature gradient is maintained; the remaining sides radiate into a medium which is at zero temperature. Initially the rectangle is at zero temperature. The problem is converted into a homogeneous one by considering a rectangle of twice the length with a uniformly distributed heat source along the center line. Problem is solved by using particular solutions given in Carslaw and Jaeger. Problem is of interest in connection with the testing of heat-insulating materials.

From author's summary by M. W. Rubesin, USA

4303. Rohsenow, W. M., and Clark, J. A., A study of the mechanism of boiling heat transfer, *Trans. Amer. Soc. mech. Engrs.* **73,** 5, 609–616, July 1951.

An analysis of the heat quantity required to form a vapor bubble in a liquid is presented. High-speed motion pictures of boiling heat transfer in forced convection showing bubbles departing from a heated surface are analyzed, and the net heat transferred to the bubbles is compared with the total heat transferred from the heated surface. It is found that the heat transferred to the moving liquid by the condensation of the bubbles is a negligible part of the total convective heat transfer. Therefore, it is proposed that the high rate of heat transfer associated with surface boiling in a subcooled liquid is due primarily to the violent agitation of the quiescent liquid adjacent to the heated surface resulting from the motion of vapor bubbles being generated there. From authors' summary by S. I. Pai, USA

4304. Chandrasekhar, B. S., and Mendelssohn, K., Subcritical flow in the Helium II film, *Proc. phys. Soc. Lond. Sec. A*, 64, part 5, 377A, 512–513, May 1951.

Authors point out that, in practically all experiments on film transport in liquid helium II, observations have been made at the critical rate of flow. An investigation is described on flow through the film at subcritical rates of flow which can be produced using the thermomechanical effect. They find that the rate of flow is proportional to the rate of heat input until a critical value is reached, beyond which the rate of film flow remains unchanged with increasing heat input.

The investigation described presents some striking evidence for the existence of a critical rate of flow in He II. This is of particular interest considering the work of Gorter and Mellink [AMR **3,** Rev. 2493] suggesting there was no critical rate. An attempt is made by the authors to reconcile their observations with the "mutual friction force" of Gorter and Mellink, while still retaining the concept of a critical rate. R. Bowers, England

4305. Tamaki, H., Studies of surface heat transfer based on boundary-layer theory (in Japanese), *Rep. Inst. industr. Sci. Tokyo Univ.* **1,** 8, 239–273, Mar. 1951.

Author considers heat transfer from a body in a fluid flow according to boundary-layer theory. Effects of gradients of both pressure and surface temperature are studied by solving the energy equation of the steady two-dimensional laminar boundary layer.

With the change of variables used by von Mises and von Kármán-Millikan for the equation of motion, the energy equation becomes $\partial T/\partial\varphi = \kappa/(\partial/\partial\psi)[u/u_1(\partial T/\partial\psi)]$, with usual notations. Using an approximate relation $u/u_1 = C(\varphi)(\psi)^{1/2}$, that is, replacing the curve u^2/u_1^2 vs. $\psi/(\varphi)^{1/2}$ obtained from von Kármán-Millikan's outer solution by its tangent at the surface, the equation is integrated into the form of a definite integral involving the surface-temperature distribution. Heat transfer from the surface is readily obtained. Various examples are worked out, the case of a flat plate with an unheated part being included. Comparing the results with the exact solutions for several simple cases, it is shown that author's solutions are sufficiently accurate for most practical purposes. Moreover, comparison of author's experiment on a flat plate having an unheated part with Schmidt's experiment on a heated circular cylinder (*Forsch. Geb. Ing.-Wes.*, 1941) shows satisfactory agreement.

Author also considers laminar boundary layer of compressible fluids along a flat plate with uniform surface temperature, taking into account the variation of physical constants and heat generated by dissipation. Equations of motion and energy are solved simultaneously by successive approximations. It is shown that, with proper choice of temperature difference and reference values of physical constants, the coefficient of heat transfer may be estimated with sufficient accuracy from that for incompressible fluid. Itiro Tani, Japan

4306. Lement, B. S., Roberts, C. S., and Averbach, B. L., Determination of small thermal expansion coefficients by a micrometric dilatometer method, *Rev. sci. Instrum.* **22,** 3, 194–196, Mar. 1951.

A method for the measurement of small thermal expansion coefficients in the vicinity of room temperature is described. The method is a modification of the normal micrometric method. Data are presented for the thermal expansion coefficients of fused quartz, commercial invar, and a special high-purity invar. Probable error in the determination of expansion coefficient over the range -40 to $+40$ C is estimated to be about $\pm 0.07 \times 10^{-6}$ C^{-1}. Warren M. Rohsenow, USA

4307. Glaser, H., The regenerator with moving heat storing bed (in German), *Forsch. Geb. Ing.-Wes.* **17,** 1, 9–15, 1951.

In the "pebble heater," a new type of heat exchanger, pebbles

(spheres of approx. $^1/_3$-in. diam) circulate in counterflow alternately through the flowing warm and cold gas. Author derives from first principles the temperature distribution over the length of the column of pebbles. It is found to be sensitive toward the thermal capacities of those quantities of warm gas, cold gas, and pebbles which enter the heat exchanger in the unit of time. If these three thermal capacities are made equal to each other, then the temperature gradients are constant; in this case there is an optimum caloric efficiency, equal to that of a recuperator of comparable length. The total weight of pebbles may be reduced appreciably by speeding the circulation. The author's results (graphs, numerical examples, and simple formulas) are useful for deciding in any special instance whether the pebble heater is superior to the traditional types of heat exchangers.

R. Eisenschitz, England

4308. Weil, L., A new method for measuring true specific heats of non-metallic materials, *C. R. Acad. Sci. Paris* **232,** 16, 1473–1474, Apr. 1951.

Author presents a method for measurement of the specific heat of semi-conductors by means of passing an electric current through thin disks of the sample material plated on its two faces with platinum. A thermocouple is sandwiched between two identically prepared disks of the sample, and measurements are made of power input, temperature rise of the disk's interface, and heating time. After calculating the heat losses from the disks and knowing the density of the material, the specific heat is determined.

Y. S. Touloukian, USA

Acoustics

4309. Chetaev, D. N., On the radiation of sound from a piston (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **76,** 6, 813–816, 1951.

Author estimates the expression $\varphi = (v/2\pi)\int_S\int r^{-1}e^{-ikr}dS$, where S is a finite region of the plane $z = 0$, this giving the amplitude of the velocity potential of sound waves from an oscillating piston. He expresses φ as the sum of an explicit term and a line integral over the boundary of S. For large k the integral is approximated to by means of asymptotic series associated with points of stationary phase. Finally, for a rectangular piston an expression is found for the radiation resistance, a numerical table being given for the case in which the piston is square.

Courtesy of Mathematical Reviews F. V. Atkinson, USA

4310. Cremer, L., The scientific principles of architectural acoustics [Die wissenschaftlichen grundlagen der raumakustik]. vol. I: Geometrical architectural acoustics [Geometrische raumakustik]. vol. III: Wave-theoretical architectural acoustics [Wellentheoretische raumakustik], Zurich, S. Hirzel Verlag, 1949, 1950; 170 pp.; xiv + 355 pp. Sw. frs. 10.50; 22.50.

Author has covered geometrical and physical room acoustics exhaustively. Volume II, "Statistical room acoustics," is yet to appear. Volume I covers physical properties of sound, geometrical laws of sound propagation, plane reflection, curved wall surfaces, whispering galleries, echos, psychological evaluation of reflected sound, the ear, distribution of sound by reflectors, sound reinforcement and sterophony, model investigations, and sound absorption.

Vol. III covers equations of sound, acoustic impedance, absorption and impedance measurements in tubes, propagation across discontinuities and through horns, reciprocity law, oblique incidence, porous sound absorbers, vibrating walls, wave theoretical treatment of rectangular rooms, propagation through absorbent ducts, and absorption in air due to viscosity, heat conduction, and molecular vibrations. The treatment in Volume III is mathematical, involving acoustic wave mechanics, but ample references to experimental methods are made. In general, both volumes cover in detail the acoustical work reported in German and other European journals. There is inadequate reference to the work of American investigators. The text would have been easier to follow if a more generous use had been made of illustrations.

Both volumes are highly recommended to workers in acoustics.

Albert London, USA

4311. West, G. D., Circulations occurring in acoustic phenomena, *Proc. phys. Soc. Lond., Sec. B,* **64,** part 6, 378B, 483–487, June 1951.

An investigation of air movements about a vibrating reed is made by means of smoke particles. Particles do not move along hydrodynamic lines but trace out small ellipses. This is a common occurrence, in many acoustic phenomena, of circulatory motion in association with vibratory motion.

E. G. Fischer, USA

4312. Pachner, J., On the acoustical radiation of an emitter vibrating freely or in a wall of finite dimensions, *J. acoust. Soc. Amer.* **23,** 2, 198–208, Mar. 1951.

Radiation field is considered as superposition of (1) field of same transmitter in infinite baffle, and (2) field with vanishing normal derivative on baffle and emitter, while in free part of baffle-emitter plane the total field must be zero. Author assumes (1) as known. Then (2) is determined by one of Rayleigh's integral equations, which is solved by a double infinite series of eigenfunctions. Final solution is written abstractly in terms of Dirac's bra and ket vectors. No numerical results are included.

C. J. Bouwkamp, Holland

4313. Lu, H., Volume viscosity and compressibilities from acoustic phenomena, *J. acoust. Soc. Amer.* **23,** 1, 12–15, Jan. 1951.

Arguing on lines similar to those of J. C. Maxwell, author formulates a differential equation supposed to govern the volume change in fluids in terms of elasticity and relaxation. A linear relation is established between the rate of expansion, the excess pressure, and its time derivative; apart from the static compressibility, two empirical constants are required, which are determined such as to fit experiments on the absorption and dispersion of sound waves in liquids and gases. Calculated values for the volume viscosity of a number of gases are found to be of higher magnitude than the ordinary (shear) viscosity. Author's method gives no account of the heat effects associated with the propagation of sound waves but may, owing to its simplicity, be useful for engineering applications.

R. Eisenschitz, England

4314. Barone, A., Optical method for measuring longitudinal and transverse ultrasonic waves in solids (in Italian), *Ric. sci.* **21,** 4, 513–515, Apr. 1951.

A method is described for measuring the velocity of longitudinal and shear ultrasonic waves in solids that employs the refraction of ultrasonic beams traveling from the solid specimen to a liquid whose acoustic parameters are known.

The refraction angles are determined on a photographic record of the ultrasonic field obtained with the schlieren method.

From author's summary

4315. Barone, A., and Nuovo, M., Ultrasonic intensity meter (in Italian), *Ric. sci.* **21,** 4, 516–518, Apr. 1951.

When an ultrasonic beam is low-frequency modulated, a

common microphone can be employed for measuring its intensity through the changes of the alternating component of the radiation pressure. An ultrasonic intensity meter based upon this principle is described. From authors' summary

4316. Sette, D., Experimental contribution to measurements of the absorption coefficient of ultrasonic waves in liquids (in Italian), *Nuovo Cim. 9th ser.*, **7**, 1, 55–63, Jan. 1950.

Paper describes an arrangement for measuring the ultrasonic absorption in liquids by an optical method, and some characteristics of the apparatus. The obtained experimental values of the absorption coefficient in some liquids agree fairly well with those found by other authors. From author's summary

4317. Carrelli, A., and Porreca, F., On the propagation of ultrasonic waves in liquids (in Italian), *Nuovo Cim. 9th ser.*, **7**, 2, 94–98, Mar. 1950.

The intensity distribution of the light lines which are formed beyond an ultrasonic grate are examined by a photographic method. Measurement is taken for liquids of different viscosity and results are obtained with the variation of this parameter. From authors' summary

4318. Porreca, F., On the distribution of intensity in ultrasonic grates (in Italian), *Nuovo Cim. 9th ser.*, **7**, 3, 171–173, May 1950.

The propagation of ultrasonics in an electrolytic solution, whose ions have different masses, produces a different ionic concentration and, hence, a difference of the refraction index, which can be valued as order of measure. However, measures of intensity of the diffracted lines made by the ultrasonic grates of such solutions do not show any difference from the theoretical results (taking measurement errors into account). From author's summary

4319. Petralia, S., Ultrasonic interferometry in gases (in Italian), *Nuovo Cim. 9th ser.*, **7**, 5, 705–714, Sept. 1950.

After a review of the principal questions connected with the scattering of ultrasonic waves in gases, paper describes a variable path interferometer which allows the determination of the propagation constants (velocity and absorption coefficient) of ultrasonic waves in gases and vapors, for frequencies between 50 and 2000 kcps; the experimental technique and the precautions to be taken in this kind of measurements are throughly treated. Notice is given of some preliminary measurements made in CO_2 and in lighting gas, from which it results that in this latter gas (that CO_2 is dispersive is known) velocity dispersion also occurs in the frequencies between 58 and 1400 kcps. From author's summary

4320. London, A., Transmission of reverberant sound through double walls, *J. acoust. Soc. Amer.* **22**, 2, 270–279, March 1950.

Present paper is an extension of a previous one on the transmission of reverberant sound through single walls [AMR **3**, Rev. 1784]. Author considers the transmission of sound through two identical mechanically uncoupled walls having a fixed spacing. The problem is treated both theoretically and experimentally. It is shown that the problem is determinate if only the impedance of a single wall and the spacing are known. As in the case of single walls, the concept of a single normal impedance is not valid. Furthermore, a wall cannot be characterized as a mass reactance; the effects of resistance (viscous damping) and flexural waves play important roles.

The attenuation of sound by the walls is materially reduced if there is any solid mechanical coupling between them. When coupling is by air alone, even small spacings can produce significant increases over a single wall. Insertion of absorbent material in the airspace produces large improvements in attenuation when the walls are relatively light, but produces little effect if the walls are heavy. Finally, a nonabsorbent cellular structure having no cell walls in a direction normal to the wall faces does not improve the attenuation of sound. Horace M. Trent, USA

4321. Fry, W. J., Fry, R. B., and Hall, W., Variable resonant frequency crystal systems, *J. acoust. Soc. Amer.* **23**, 1, 94–110, Jan. 1951.

The resonant frequency of a crystal system is dependent on the dimensions of the liquid medium backing the crystal. When the free resonant frequency is known, the dimensions of the backing necessary to obtain a given frequency are calculated for the lossless system (imaginary acoustic impedance). Several loss factors are considered and expressed in a "quality-factor." The theoretical results are compared with experiment. W. H. Muller, Holland

4322. Rozhdestvenskiĭ, B. L., Waves in a plane horn (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **77**, 2, 221–224, Mar. 1951.

The method of conformal representation is applied to the problem of excitation of a plane horn by electromagnetic waves; a differential equation is obtained which can be solved if a few simplifying assumptions, in no way inconsistent with actual conditions, are made. Maxwell's equations are expressed in terms of dimensionless curvilinear othogonal coordinates, and the coefficient of reflection is deduced for the plane horn acting as (1) a radiator and (2) a receiver.

For horns of small angular aperture, the fundamental of the incident waves is reflected and, as the aperture is increased, upper harmonics are successively reflected. The straight horn possesses the best characteristics as a radiator, and the horn turned back through almost 180° is the worst. The method described has very general application. Marie Goyer, England

4323. White, J. E., Some effects of vibration on X-ray diffraction by crystals, *J. acoust. Soc. Amer.* **23**, 1, 16–18, Jan. 1951.

The theoretical calculation of the amplitude function for elastic vibrations of bodies under usual conditions is very difficult. Therefore, exact experimental methods are the more desirable. The old method of using a dust pattern is rough and only applicable to the detection of internodal distances. A more successful method was developed by the use of a pick-up. However, a true picture of the amplitude function cannot be brought about in this way because the pick-up is coupled to the object to be examined. Instead, if the vibrating surface is searched by an optical beam there is no reaction to the object. Now, in the case of a crystal, a picture may be obtained by diffraction of x rays, since Bragg reflection is greatly enhanced by vibration or other inhomogenous strain. This is the principle which author uses.

The equipment consists of an x-ray tube, a piezoelectric transducer, and a screen. Molybdenum radiation is filtered by a zirconium foil, limited by slits and directed under acute angular incidence on the surface of the crystal which in turn is cemented to the transducer. The reflected beam falls under the same incidence upon the screen which is an x-ray sensitive film. During the exposition, the crystal is rotated through about 2°, the screen at twice the angular rate of the crystal, to give all regions an equal chance to diffract. Using three series of illustrations, author discusses what may be read in the photography: (1) Patterns for different flexural resonances of a square plate. Pictures show as

many details as are visible in dust patterns and, besides, a more subtle gradation. However, proportions are only approximated. No correlation with dust pattern could be found. (2) Pattern of a fixed vibration configuration under different orientation arranged by rotation round the normal to the surface. Picture is preserved generally but blackening is altered. This fact suggests that amplitude itself is not responsible for the observed increase in x-ray reflection, but instead the local curvature of surface, that is, the second partial derivative of the amplitude function. (3) Patterns of a clamped-free bar vibrating in different modes. In this case, the theoretical amplitude function depends only on the distance along the bar. In this way the suggestion as to the influence of curvature could be examined, and indeed it was found all right. Peter-Paul Heusinger, Germany

4324. Beyer, R. T., The contribution of thermal conductivity to the ultrasonic absorption coefficient in liquefied gases, *J. chem. Phys.* **19,** 6, 788–789, June 1951.

Author presents results of his computation of the contribution to the ultrasonic absorption in liquefied gases due to thermal conductivity. He uses known experimental values or the approximate theoretical values of the thermal conductivity of the liquified gases. He compares his results with Galt's experimental data and states that satisfactory agreement is obtained. Tables I to III present data for liquid argon, oxygen, nitrogen, and hydrogen.

W. H. Pielemeier, USA

Ballistics, Detonics (Explosions)

4325. Drenick, R., The perturbation calculus in missile ballistics, *J. Franklin Inst.* 251, 4, 423–436, Apr. 1951.

The calculus of perturbations, as used in exterior ballistics, is modified for application to missiles which experience thrust, drag, and lift forces. The effects of disturbing factors upon a trajectory are investigated and certain principles of guidance and control are established analytically.

From author's summary by A. Petroff, USA

4326. Morris, G., The reduction of ground vibration from blasting operations, *Engineering* 169, 4395, 430–433, Apr. 1950.

Experiments of testing the effect of short-delay firing on ground vibrations are described. Time intervals of 25 millisec were obtained by using, alternatively, special short-delay detonators made by the Nobel Division of the Imperial Chemical Industries, and a sequence switch consisting of a rotating contact driven by a clockwork motor. Up to 14 circuits are closed in order, at the required time intervals.

Tests have been made in a wide variety of quarries and opencast coal sites. Vibrograph records of building vibrations have been taken both for delay shots and for simultaneous shot-firing at the same site. Typical vibrograms are reproduced. The two principal parameters of a vibrogram, the maximum amplitude and the average frequency of the vibration, are set out in a table containing 31 tests.

It has been found that the average frequency, which is a characteristic of the ground, is practically the same for instantaneous and delay firing.

The ratio of the maximum amplitude measured with delay blasting to the maximum amplitude from the instantaneous firing of the weight of explosive used in each hole of the delay blast, is derived from the records of simultaneous firing at the same site, using the formula $a = (A.E^{1/2})/d$, where A is a site factor and a is the maximum amplitude of ground motion caused by firing E pounds of explosive at a distance of d ft. This ratio is of the order of unity; i.e., the effect on a building of firing a number of equal charges at 25-millisec intervals is not very different from what would have been caused by firing only one such charge. This ratio does not seem to depend to any great extent on the number of charges fired, but there is evidently a pronounced site effect.

When these amplitude ratios are plotted against the ratios ϕ of the delay time to the natural period of the site, the data lie quite close to a smooth curve of the type suggested by a theoretical treatment given in an appendix.

It was first thought that the more regular time intervals given by the sequence switch, compared to those given by short-delay detonators, might show reduced amplitude of ground vibration, but there seems to be no difference in performance in the two methods of delay firing. W. Weibull, Sweden

4327. Frye, W. E., On the accuracy of the long-range ballistic rocket, *J. appl. Phys.* **22,** 5, 585–589, May 1951.

The accuracy of a B-2 type ballistic rocket, which is guided to the termination of propulsion and is in free flight thereafter, is investigated. The effect of the rotation of the earth and the reentry into the atmosphere on the accuracy is considered and the errors due to lack of thrust control are analyzed.

From author's summary by Glen Goodwin, USA

4328. Boden, R. H., Heat transfer in rocket motors and the application of film and sweat cooling, *Trans. Amer. Soc. mech. Engrs.* **73,** 4, 385–390, May 1951.

Paper presents results of heat-transfer tests of liquid-fuel rocket with and without internal film cooling. Local heat-transfer rates were measured along the length of the rocket. It was found that small amounts of injected coolant had a marked effect on heat-transfer rates. Analysis of the data in terms of the usual heat-transfer parameters is not attempted, presumably due to lack of knowledge of combustion temperature.

Heat-transfer processes in rockets are analogous to those occurring in internal-combustion engine cylinders, and reviewer believes experimental work in this field would be aided by use of Pinkel's concept of "effective gas temperature" [cf. *NACA tech. Rep.* 612, 1938, and *NACA tech. Rep.* 853, 1946].

Jackson R. Stalder, USA

Soil Mechanics, Seepage

(See also Revs. 4140, 4142, 4164, 4326)

4329. Rapoport, L. A., and Leas, W. J., Relative permeability to liquid in liquid-gas systems, *J. Petr. Technol.* **3,** 3, 83–95, Mar. 1951.

The term *relative permeability* denotes the constant of proportionality K_L in the generalized form of Darcy's law: $v_L = k\ K_L/\mu_L$ grad P_L; k is specific permeability of the porous medium, μ_L viscosity, grad P_L the potential gradient, i.e., the relation between effective and specific permeabilities of a three-phase system (gas, liquid, solids). Using Koženy's formula for specific permeability, authors deduce theoretical relations for this system, considering distribution of the three phases to be governed by surface-energy relationships and the dynamic equilibrium established between the interfaces. Generalizing the concept of laminar flow, results pertain to parallel flow when the phases move in an independant manner through two interwoven but distinct networks of channels. Corresponding interfacial areas have to be evaluated; by means of capillary pressure limiting values they are computed. Based on theoretical considerations, authors prove the relative permeability to be function of

the degree of saturation. Data computed by the formulas obtained compare very satisfactorily with experiments carried out with an ingeniously constructed apparatus.

A. Kézdi, Hungary

4330. Rowe, P. W., The distribution of lateral earth pressure on a stiff wall due to surcharge, *Civ. Engng., Lond.* **45**, 531, 532; 590–592, 654–657; Sept., Oct. 1950.

Author reports results of 150 tests made in a box 2 ft × 2 ft × 2 ft using dry sand to determine magnitude and distribution of pressures exerted on one side of the box due to loads applied to sand surface. Surcharge loads up to a maximum intensity of 4 psi were used. Effect is investigated of position of surcharge with respect to one side of the box (a 2 ft × 2 ft × $^3/_8$-in. steel plate simulating vertical retaining wall) on which pressures were measured. Residual effects as well as effects of wall movements are reported. Experimental results are compared with those obtained from theory.

Following results are observed: (1) Lateral pressure increases linearly with surcharge pressure. (2) Distribution of vertical pressure in soil mass as measured is in reasonably good agreement with that computed by the Boussinesq formula; presence of vertical wall seems to have little effect on vertical pressure in soil mass. (3) As distance from wall to a loaded strip increases, resulting total lateral force on wall decreases and center of pressure falls from point near top of wall toward center of wall. (4) With increasing width of loaded area from wall edge, pressure distribution diagram approaches rectangular shape. When width of loaded area equals height of wall, intensity of lateral pressure equals approximately 0.3 surcharge pressure. (Ratio of lateral to vertical pressure, under this condition, approximately equal to $K_a = \tan^2(45° - \phi/2)$. (5) No essential change in magnitude and distribution of lateral pressure on wall occurs when wall is permitted to yield by rotation about toe. When wall is rotated about top, total load on wall does not change significantly but point of application of resultant load shifts toward top of wall. (6) When surcharge load is removed, residual pressure remains. When surcharge area extends from face of wall a distance equal to height of wall, residual pressure equals 50 to 60% of pressure existing before surcharge load was removed. Where surcharge acts over a smaller area, residual lateral pressure is a smaller percentage of original pressure.

Data submitted by author adds to meager store of experimental evidence on surcharge effects.

R. E. Fadum, USA

4331. Polubarinova-Kochina, P. Ya., and Falkovich, S. B., Theory of filtration of liquids in porous media, Advances appl. Mech. **II**, 153–225, 1951. [New York, Academic Press, Inc.]

See AMR **2**, Rev. 280.

4332. Minami, K., Research on shell foundations, *Memo. Fac. Sci. Engng. Waseda Univ., Tokyo*, no. 14, 110–113, 1950.

A method is proposed for increasing the bearing capacity of foundations by use of a thin hollow shell sunk into the ground. The load is applied to part of the enclosed surface. Natural soil inside the shell may be removed and replaced with sand or gravel. Design is based upon Terzaghi's simplified method of determining the bearing capacity of shallow footings. A large number of laboratory tests were made to determine the bearing capacity of footings using various shapes and sizes of bearing plates and shells. From this data, curves are developed into a so-called "bearing capacity increase coefficient" chart to be used in design. Reviewer feels that caution should be exercised in extrapolating from curves based on small laboratory tests (bearing plates of 50 and 100 sq cm) to full size footings.

A number of field tests were made using larger shells and plates, but only general results are indicated (no data). Author states that charts can be constructed under field conditions; also that the bearing capacity in ordinary loam can easily be increased four or five times by using proper shells.

Phil M. Ferguson, USA

4333. Kjellman, W., Testing the shear strength of clay in Sweden, *Géotechnique Lond.* **2**, 3, 225–232, June 1951.

Paper outlines variety of methods for determining laboratory and in-place strengths. For routine direct shear testing, sample is hermetically sealed in a flexible tube. Shearing load is applied tangentially to the top surface, producing an angular deformation of the sample. Claims for this apparatus are a more uniform stress distribution than in conventional direct shear devices, as well as better moisture control and freer consolidation. For research on direct shear testing, a second more complicated device is used, which produces a condition of pure shear in the sample.

Other interesting tests include a triaxial, in which the three principal stresses applied to the sample may be varied independently; a laboratory-scale cone-penetration test; an in-place-type vane test in which resistance to turning measures shear strength. Author states vane test gives most reliable results, primarily because it involves no sampling operation and resulting decrease in strength.

Philip P. Brown, USA

4334. Casagrande, A., Notes on the design of earth dams, *J. Boston Soc. civ. Engrs.* **37**, 4, 405–429, Oct. 1950.

Presented originally as a public lecture, author's paper gives a critical discussion of factors involved in earth-dam design. The first half presents comments on the shear strength of soils and its use in the Swedish method of stability analysis. Author decries the improper use of routine test results for cohesion and internal friction and emphasizes the determination of shearing strength corresponding to state of consolidation of soil. A pertinent illustration of effect of change of volume is given. Four methods of analyzing slope stability are discussed, but it is emphasized that these are just plain mechanics and the accuracy depends upon the use of proper shear-strength values.

The second half describes causes of earth-dam failures and discusses two types: (a) flow slides, and (b) piping resulting from rupture due to irregular settlement. One of the striking examples presented tells about a flow slide in loessal soils where, typically, the fluid involved in liquefaction is air rather than water. Two examples of piping failures point out that, if there are conditions leading to sharp differential settlements, special provisions should be made to assure that any break will be self-healing.

George J. Tauxe, USA

4335. Lambe, T. W., Soil testing for engineers, New York, John Wiley & Sons, Inc.; London, Chapman and Hall, Ltd., 1951, ix + 165 pp. \$5.

This textbook for teaching laboratory soil testing presents recommended methods for soil description and care of samples; specific gravity; liquid, plastic, and shrinkage limits; grain-size analysis; compaction, permeability, capillarity; consolidation; direct and triaxial shear for cohesionless and cohesive soils; and the unconfined compression test. For each test there is a list of equipment, numerical example using data forms, and well-illustrated discussion of procedure and results with references. Appendixes include useful tables, data on proving rings, and derivation of equations. While the preface warns against blindly following test procedures, standard ASTM procedures are not given their due consideration.

Edward S. Barber, USA

4336. Polubarinova-Kochina, P. Ya., On nonsteady flow of soil waters (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **75**, 3, 357–360, Nov. 1950.

A numerical procedure for studying flow of ground water is based upon the formula

$$N = km^{-1}[\cos(n, y) - \gamma^{-1}\partial p/\partial n]_S$$

where N is the velocity of translation of a point on the free surface, k, m, and γ are filtration constant, porosity, and density respectively. Having the pressure distribution at a given time $\partial p/\partial n$ can be approximated, and a new position for the free surface found approximately. Pressure distribution is then redetermined, and the process repeated. No examples are given.

R. E. Gaskell, USA

4337. Huisman, L., and Kemperman, J., Pumping of underground water under pressure (in Dutch), *Ingenieur* **63**, 13, B. 29–B. 35, Mar. 1951.

A horizontal layer of sand is bounded by two layers of clay (with lower permeability) and is pierced by a well penetrating the whole sand thickness. By pumping, this well supplies water steadily. Authors introduce the permeability coefficients of the considered layer of sand and of the sand under the lower layer of clay. They take into account the resistance of each of the two layers with lower permeability against the vertical motion of the water. Formulas are deduced for the resulting pressure of the ground water in each of the two layers of sand. The lower layer of clay has a considerable influence even if it is of small thickness. In the case of seepage into a polder, the corresponding influence is much smaller.

L. J. Tison, Belgium

4338. Brown, H. W., Capillary pressure investigations, *J. Petr. Technol.* **3**, 3, 67–74, Mar. 1951.

A comparison is given, by means of diagrams, of static capillary pressure measurements on limestone and dolomite cores performed by: (a) the restored-state method; (b) the mercury-injection method; and (c) dynamic capillary pressure measurements by means of the Hassler technique. The conversion factor required to give good agreement between methods (a) and (b) was found to be constant for each core sample, but was not constant for all samples. For limestone cores this factor averaged 6.4, and for sandstone cores, 7.2.

The static and the dynamic methods gave nearly identical results. The conclusion is drawn that static capillary pressure data may be used in dynamic problems of fluid flow. Data are reported which justify the use of the J-curve for correlation purposes in respect to samples of specific lithologic types from the same formation.

Gregory P. Tschebotarioff, USA

Micromeritics

4339. Winslow, F. H., and Matreyek, W., Particle size in suspension polymerization, *Indust. Engng. Chem.* **43**, 5, 1108–1112, May 1951.

Control of size and geometrical form of density cross-linked hydrocarbon polymers yields fluid spherical powders useful as dielectrics and in rheological studies.

With polyvinyl alcohol as a stabilizer it was found that, under comparable conditions, (a) high molecular weight grades, (b) partially hydrolyzed grades, and (c) high stabilizer concentrations were associated with spheroids of lower mean diameters (down to stabilizer concentrations of about 0.1%). At concentration limits where suspending action begins, it was found that the number of polyvinyl alcohol molecules present was important; for equal weight concentrations of about 0.005%, low molecular weight polymer (19,000) gave stabilized but large spheres whereas high molecular weight polymer (95,000) was ineffective. Close to the maximum possible yield of well-formed spheroids was reproducibly obtained in narrow size distribution and with average diameters ranging from 5 microns to several mm.

The results have a bearing on polymer forms used in ion-exchange resins.

From authors' summary by S. G. Ward, England

Geophysics, Meteorology, Oceanography

(See also Revs. 4060, 4085, 4200, 4206, 4228)

4340. McVittie, G. C., Coordinate systems in dynamic meteorology, *J. Meteor.* **8**, 3, 161–167, June 1951.

Paper contains a general survey of the methods of setting up coordinate systems in dynamic meteorologies and a description of certain systems derived from geometric considerations. Equations of motion and continuity are given explicitly, and general formulas for the divergence and rate of change of vorticity are listed. Spherical polar coordinates lead to the introduction of intrinsic coordinate systems, in which the atmospheric motion itself defines the coordinate system. A brief discussion is devoted to the relationship between the size of the atmospheric motion and the coordinate system which best describes it, to gradient and geostrophic winds, and to the Rossby formula for potential vorticity.

From author's summary by Horst Merbt, Sweden

4341. Ketchum, B. H., The exchanges of fresh and salt waters in tidal estuaries, *J. mar. Res.* **10**, 1, 18–38, June 1951.

Paper presents an empirical theory which describes the exchanges between various parts of an estuary as a result of tidal oscillations. The parameters used in the calculation are the mean range of tides, river flow, and topography. Theory is based on the assumption that complete mixing occurs in volume segments defined horizontally by the width of the estuary and the average excursion of a particle of water on the flood tide, and vertically by the depth of the mixed layer.

Calculated distributions of river water are in close agreement with measured values for three very different estuaries: Raritan River and Bay, New Jersey, Alberni Inlet, British Columbia, and Great Pond, Massachusetts.

John H. Carr, USA

4342. Miyazaki, M., Lateral mixing effect on the distribution of ocean currents, *Oceanogr. Mag. Centr. Meteor. Observ. Tokyo*, **2**, 3, 117–121, Sept. 1950.

Paper gives some numerical results of the relation between a hypothetical ocean circulation and the corresponding wind circulation which drives it, for different values of lateral mixing coefficient in the ocean. The theory has been given elsewhere [Miyazaki, M., *Mem. K. M. O.* **8**, 41–43, 1950]. To understand the meaning of the symbols and the results, the reader must refer to the above reference. Author employs a different approach to the problem of circulation by starting with a presumed ocean circulation and evaluating the wind field therefrom. The effect of lateral mixing and of the spherical earth are taken into account. Problem is solved for a spherical sector of the earth in the northern hemisphere. Results of the deduced wind circulation for four different values of the lateral mixing coefficient ranging from zero to infinity are given in graphical form and are compared with a climatological mean isobaric chart over the Pacific. Author concludes best agreement for case of mixing coefficient of 6×10^{13} c.g.s. The latter value is the greatest yet proposed. There is,

however, some doubt in regard to the conclusions, if any, that one can draw from this study for the real ocean-atmosphere system. This is because of the oversimplified current system which author assumes. The westward intensification and equatorial counter current, both dynamically important features of the circulation, are not taken into account in the analysis.

Robert O. Reid, USA

4343. Roseau, M., On the undulatory motions of the ocean on a seashore (in French), *C. R. Acad. Sci. Paris* **232**, 3, 211–213, Jan. 1951.

The problem is identical with that investigated by E. Isaacson [AMR **4**, Rev. 2371]. The potential function $\phi = e^{i\sigma t}\, \varphi(x, y)$ is determined by a direct method of solution for an arbitrary angle α of the plane sloping bottom. Walter Wuest, Germany

4344. Roseau, M., On the undulatory motions of the ocean on a seashore (in French), *C. R. Acad. Sci. Paris* **232**, 4, 303–306, Jan. 1951.

Following the method mentioned in preceding review, the potential function $\phi = e^{i\sigma t}\varphi(x, y)$ is determined for an angle $\alpha = \pi/2n$ (n entire) and the potential function $\phi = e^{i(kz+\sigma t)}\varphi(x, y)$ (waves of finite length of crest) for $\alpha = \pi/4$.

Walter Wuest, Germany

4345. Lesser, R. M., Some observations of the velocity profile near the sea floor, *Trans. Amer. geophys. Un.* **32**, 2, 207–211, Apr. 1951.

Author lowered a current meter to measure horizontal velocities from an anchored ship to the sea bottom. The depth was about 50 m, below that of wave action. Instrumentation is described. A total of 18 observations was made over three types of sea floor. Site of observations and duration of experiments are not given.

The meter recorded drift at 20, 40, 80, and 160 cm above the bottom. All velocity profiles were logarithmic and the direction of drift changed little with height. The velocity distribution over gravel sand and mud sand corresponded to that to be expected over a rough boundary surface. It corresponded to flow over a smooth boundary surface over mud. Herbert Riehl, USA

4346. Lettau, H., Theory of surface-temperature and heat-transfer oscillations near a level ground surface, *Trans. Amer. geophys. Un.* **32**, 2, 189–200, Apr. 1951.

A mathematical treatment of the heat balance at the earth's surface and the related temperature distributions in the air and ground. The conductivity of the ground is assumed constant with depth and time; the coefficient of eddy diffusivity of the air is assumed to vary linearly with height but to be constant with time. The problem of cooling due to evaporation at the earth's surface is mentioned but not treated fully. Diurnal and annual fluctuations of temperature and heat flux are assumed to be sinusoidal, and amplitude and phase relationships between the variables are established. Frequent reference is made to a previous paper of the author [*Geophys. Research Papers*, no 1, *Air Force Cambridge Res. Lab.*, 1949]. Limited experimental verification of the results is given, based on work by Albrecht [Reichsamt f. Wetterdienst, Berlin, *Wissenshaftl Abh.* **8**, no. 2, 1940]. Reviewer believes that the major aspects of dry micro-climates may be explained by author's paper, but the questions of evaporation and variable (with time) eddy diffusivity will probably have to be considered if the theory is to be extended.

Franklin I. Badgley, USA

4347. Bikov, V. D., Hydrometry (Gidrometriya), Leningrad, Gidrometeorol Izd., 1949, 463 pp. $2.25.

A new textbook on water measurement for Russian hydrological schools and universities. This field is very significant in Russia, which has about 5000 gaging stations, even in the polar regions of Siberia. Book contains: Organization of the water investigations in Russia, water stage observations, measurement of depths, measurement of velocities and direction of current, determination of discharges and their calculation, relation between the stage and the discharge, investigation of the solid runoff and sediments, special observations (quality of water, waves, thermic regime). Instruments and methods of their rating are considered; also their application and care. Textbook is very extensive and gives a good picture of the modern development of hydrometry in Russia. Besides several Russians, Americans, including Stevens, Price, Stout, Haskell, Hoff, Cole, Bakhmeteff, Allen, and Taylor are mentioned. However, the names of several Russian authorities in hydrology, e.g., V. G. Gluchkov, E. V. Oppokov, are omitted, although instruments of their invention are described and used. Foreign literature is not covered in the bibliography and many original pictures from a treatise by the reviewer are used without referencing. Steponas Kolupaila, USA

4348. Sutton, O. G., The dispersion of hot gases in the atmosphere, *J. Meteor.* **7**, 5, 307–312, Oct. 1950.

A semi-empirical equation is obtained for the diffusion of hot gas emitted into a calm atmosphere from a point source. Assumptions include an expression for rate of entrainment of air by the jet and the applicability of the theory of isotropic turbulence. The equation indicates the temperature above the source varies logarithmically with height. Results are checked against experimental data obtained by Schmidt. The problem of hot gases emitted into an atmosphere with uniform horizontal wind is briefly considered by superimposing a basic current on the previous solution. Reviewer believes the equations are of value as crude solutions when atmosphere exhibits no thermal stability or instability, as assumed by author. However, the appearance of precision given by writing empirical coefficients to two decimal places is unfortunate and misleading. Furthermore, since neutral stability seldom exists in the atmosphere, the equations must be used in practice with extreme caution.

George S. Benton, USA

Lubrication; Bearings; Wear

4349. Schwaben, R., and Umstatter, H., Structural mechanics of rheonomic systems. On the rheological theory of bearing friction (in German), *Kolloid Z.* 118, 1, 38–55, July 1950.

The classical theory of journal lubrication is treated with the assumption that the viscosity of the lubricant used diminishes when the velocity gradient increases. Pressure distribution, the curve described by the center of the journal, and the coefficient of friction are calculated and compared with the case when a pure viscous lubricant is used. A table containing the values of the integrals used in the classical theory of journal lubrication is given at the end of the paper. D. De Meulemeester, Belgium

4350. White, H. S., and Zei, D., Static friction tests with various metal combinations and special lubricants, *J. Res. nat. Bur. Stand.* **46**, 4, 292–298, Apr. 1951.

Inclined-plane apparatus showed that lubricants with the oxidized petroleum compound and those with a silicone grease gave slightly less friction than Navy symbol 2110 reference oil. Mineral grease with dry molybdenum disulfide and lubricants with graphite or molybdenum disulfide gave about 50% less friction. Heat-treated stainless-steel combinations gave lowest

friction, while combinations with cast iron or aluminum alloy gave relatively high friction. Lapped and polished surfaces gave less friction than ground surfaces. C. R. Freberg, USA

4351. **D'yachkov, A. K., Application of the theory of effective loads for the calculation of bearings of reciprocating engines** (in Russian), *Izv. Akad. Nauk. SSSR Otd. tekh. Nauk*, no. 11, 1615–1644, Nov. 1950.

Paper presents derivation of expressions for effective impulse, local pressures, and load-carrying capacity of journal bearings of internal combustion engines. Instantaneous loadings of crank shaft and main bearings due to inertia of rotating and reciprocating parts are analyzed. Location of oil holes and critical speeds are discussed. A. Petroff, USA

4352. **Johnson, R. L., Peterson, M. B., and Swikert, M. A., Friction at high sliding velocities of oxide films on steel surfaces boundary-lubricated with stearic-acid solutions**, *Nat. adv. Comm. Aero. tech. Note* 2366, 35 pp., May 1951.

A study was made to determine the effectiveness of stearic acid as an additive in cetane for lubricating steel surfaces at high sliding velocities (75 to 7000 fpm) and heavy loading (initial Hertz surface stress, 108,000 to 194,000 psi). This work is applicable to the case of aircraft engine lubricants which are compounded to operate satisfactorily at low temperatures, but must be made to provide adequate lubrication at engine-operating temperatures.

Tests show that the presence of a heavy Fe_3O_4 film on the steel surfaces improves the lubrication effectiveness of stearic acid and cetane, especially at high velocities and heavy loads. Fe_2O_3 films do not exhibit the same beneficial effect. Authors find that stearic acid is oxidized at temperatures above 130 C, reducing Fe_2O_3 to the more desirable Fe_3O_4. A rubbed film of pure stearic acid is effective in inhibiting lubrication failure on clean steel and oxide-coated surfaces. Eugene M. Simons, USA

4353. **Soda, N., and Miyakawa, Y., Friction of mono- and multi-molecular layers** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **4**, 7–8, 176–190, July–Aug. 1950.

Kinetic frictions of mono- and multi-molecular layers of stearic acid and of barium stearate were measured for various conditions of loads, temperatures, curvatures of sliders, and combinations of rubbing materials. The Langmuir-Blodgett technique was used for deposition of molecular layers on plate (*Y*-deposition). Friction decreases with the increase of number of layers in accordance with the previous observations by other investigators, but the effects of curvature of the slider and the combination of rubbing materials are not found to be negligible. Two regimes in boundary friction reported before [title source, **1**, 50–53, 1947] are also observed, the transition pressure being 150–200 kg/mm² and the film thickness for the second regime estimated to be of one or three molecular layers. The mechanical transition temperatures for various thicknesses of molecular layers were measured by stick-slip method, and are found to increase with the number of layers, saturating at the thickness where the friction attains its lowest value. Isao Imai, Japan

Marine Engineering Problems

4354. **Bonebakker, J. W., The application of statistical methods to the analysis of service performance data**, *Shipbuilder* **58**, 512, 382–385, May 1951.

See AMR **4**, Rev. 4037.

4355. **van Lammeren, W. P. A., Results of methodical test series extended to 2- and 5-bladed propellers** (in Dutch), *Schip en Werf* **18**, 8, 155–162, Apr. 1951.

Report is based on a paper by Troost [AMR **4**, Rev. 3103]. The influence of number of blades on efficiency and on optimum diameter is discussed. In general, both these quantities decrease with increasing blade number. Hermann W. Lerbs, USA

4356. **Rösingh, W. H. C. E., Ships' speed on the measured mile** (in Dutch), *Schip en Werf* **17, 18**; 26, 3; 566–569, 56–59; Dec. 1950, Feb. 1951.

Two known methods are dealt with, both with assumptions that current as function of time is a well-faired curve and that there are no waves and no wind. On many ships' trials none of these assumptions holds, and it would be worth-while to estimate influence of irregular current, wind, and waves. First method is "means of means," which does not always give correct results, even with the assumptions made, but is much better than arithmetic mean. Author shows that method is correct if $\Sigma(-1)^i \binom{n-1}{i-1} v_i = 0$ from $i = 1$ to $i = n$, where n is number of runs over mile, v_i mean current during i-th run, and $\binom{n-1}{i-1}$ binomial coefficients. If v_i can be expressed as a polynomial of p-th degree, and the time interval between runs is always the same, method is correct if $n > p + 2$. p should not be less than 4.

Second method is also based on possibility of expressing current as a function of time as a polynomial of p-th degree. One obtains n linear equations with n unknowns, viz., speed of ship and $(p + 1)$ coefficients of polynomial.

It is shown that runs with different speeds can be included in the same system of equations. If m different speeds are measured (to obtain part of speed/power curve), degree of polynomial $p = n - m - 1$; e.g., with 10 runs at four different speeds, $p = 5$.

Article concludes with explanation of Prof. Prescott D. Crout's remarkable method of solving systems of linear equations.

Georg Vedeler, Norway